# RISK TRANSFER AND CAPITAL ADEQUACY

## Manual 5

WORLD BANK GROUP

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IN PARTNERSHIP WITH DONORS

WORLD BANK GROUP

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# Preface and Acknowledgements

Five manuals were prepared by IFC for the development of agri-insurance markets where the public and private sectors work together in a partnership (PPP). The manuals are designed to strengthen the capacity of the government and market players to effectively design agri-insurance products, both traditional indemnity and index, introduce them to the market, and build sales. The manuals are designed to be succinct yet at the same time sufficient to create the technical and administrative foundation for a modern agri-insurance system, and to allow programs in early stages of development to properly plan the required system. Finally the manuals are designed to train practitioners, to build local capacity for skills that are required to start the program, and to enable the program to grow over time.

The principle author of the manuals is Professor Myles Watts, University Professor, Lead Actuary at Watts & Associates, Member of the Board at the Federal Agricultural Mortgage Corporation, and 5th Generation Montana Farmer. Watts and Associates designed and launched numerable agri-insurance products in North America, frequently consults for the major reinsurers, and supports insurance programs around the world. They have established their own index insurance company, eWeatherRisk. The manuals incorporate practical lessons learned over the past 40 years.

The development of the manuals was a joint activity of the Ukraine Agri-Insurance Project (2007-2015), IFC's Global Agri-Finance Team, and the Global Index Insurance Facility (GIIF) (2009 to present). Dr. Gary Reusche led the Ukrainian project, served as a technical specialist on the global agri-finance team, and as a member of the GIIF technical committee and core management team. Agri-insurance development is closely linked to agricultural finance and value chains and they are effectively developed in unison.

The manuals result from training workshops developed by the agri-insurance project in Ukraine and globally by GIIF technical experts. The entire agri-insurance team in Ukraine made practical contributions to the manuals, with special recognition due to Victoria Yakubovich for collecting, organizing and preparing the initial drafts and Andrey Zaripov a member of the GIIF team for helping to develop the reinsurance and cash flow models. The project team included experts from the Alberta (Canada) provincial agri-insurance program, in particular Richard McConnell, who contributed his experience and expertise to the training activities.

Peer review and Spanish language translations of the manual resulted from IBRD consultants in Central and South America, especially Pablo R. Valdivia Zelaya and Roberto Dario Bacchini.

The team is grateful to Professor Gary Brester for his practical contributions and editing of the manuals and Olesya Zhuchenko for coordinating the publication of the manuals, working with designers to plan the layout, and printing.

Finally support for the manuals was provided by the Canadian government, and the Global Index Insurance Facility (GIIF) lead by Gilles Jacques Galludec (Program Manager) funded by the European Union, Japan and the Netherlands.

# Acronyms

IFC – International Finance Corporation

IIARM – International Institute for Agricultural Risk Management

DFATD – Foreign Affairs, Trade and Development Canada

CL – Confidence Level

GAO – Government Accountability Office, United States

GIIF – Global Index Insurance Facility

HA – Hectares

LGD – Loss-Given Default

SQL – Structured Query Language

UAH – Ukrainian Hryvnia, money unit of Ukraine

UI – Unexpected Indemnities

UL – Unexpected Losses

VaR – Value-at-Risk

# 1.0. Introduction

Agricultural production is inherently subject to a variety of risks because management decisions or states-of-nature often generate future outcomes (either favorable or unfavorable) that cannot be predicted with certainty. The variability of these outcomes represents risk. Some risks are managed through production and financial decision-making, while others are simply accepted as business expenses. In addition, some risks can be managed through a variety of contractual and insurance-related products.

On average, financial activities with low levels of risk are associated with lower potential returns. Low-risk investment actions tend to generate very small returns. Conversely, high levels of financial risk are generally associated with high expected returns. However, the risk/return trade-off does not mean that accepting high levels of risk *guarantees* higher returns. Rather, high levels of risk provide the *possibility* of high returns and vice versa. Individuals and firms must be compensated for accepting higher levels of risk with at least the potential to receive higher returns. An individual's willingness to accept risk depends primarily on the willingness and ability of that individual to bear risk. The degree of risk aversion (or risk acceptance) depends on many factors, including personality traits, experience, financial reserves, and relationships among business partners.

Risks associated with agricultural production ultimately impact the financial viability and sustainability of farms and ranches. Agricultural production is often coincident with high short-term credit risk because of a combination of high fixed costs, weather variability, disease, and variations in cash receipts. In an average year, annual net farm revenues may be sufficient for agricultural producers to make principal and interest payments on debt and realize profits, but across-year revenue variability may cause farm businesses to fail because of periodic inabilities to service debt obligations. Hence, whether an agricultural producer self-insures or uses formal mechanisms for transferring risk to others, risk is a cost that must be effectively managed.

Agricultural production risks also impact the viability of businesses that supply agricultural credit and insurance services to agricultural producers. Agricultural finance companies must account for potential reductions in debt repayment as a result of agricultural production risks. Hence, they must either maintain adequate capital reserves or pay fees to transfer this risk to other entities. The term "capital" refers to the equity available to cover unexpected losses (e.g., a revenue shortfall or cash outflow) so that all counterparty liabilities can be fully reimbursed. Likewise, agricultural insurance providers must retain adequate capital levels for servicing potential indemnity payments that result from agricultural production risks. Alternatively, firms may participate in a variety of risk-sharing arrangements with other companies or with governments.

All firms must evaluate the relative merits of three general risk-management strategies: (1) avoidance, (2) absorption, and (3) transfer. First, a firm can opt to avoid risk, reducing or eliminating risky opportunities. For example, a lending agency can choose to not offer credit to high-risk producers. Of course, insurance firms exist to accept risk from others, but they can choose to provide insurance to less-risky enterprises or in less-risky regions. Second, a firm can decide to absorb or accept risk that has been incurred. In these cases, capital adequacy is an

important aspect of risk management. Third, a firm can decide to transfer risk to others. This is essentially what happens when an agricultural producer cedes risk to an insurance company in exchange for paying an insurance premium. Likewise, credit or primary insurers can transfer risk to other companies in exchange for a fee. This process is generally termed "reinsurance."

## 1.1. Credit and Insurance Relationships

Although a variety of approaches exist to manage risk, each involves transaction costs and risk premiums paid by those seeking to mitigate risk to those willing to accept additional risk. Transaction costs and risk premiums can be incorporated into: (1) interest rates, (2) insurance, and (3) other instruments. These approaches can be viewed as "options." Options are financial instruments that contractually specify the events that trigger offsetting payments.

Loan default risk can be incorporated into operating, intermediate, and real estate loan interest rates. The advantage of incorporating risk premiums into interest rates (rather than other instruments) is that transaction costs are reduced because only two entities (a borrower and a lender) are involved. That is, the costs of risk transfer increase as additional entities are included. For example, a third party (e.g., an insurance company brokerage firm) requires substantial information about borrower risks, increasing total transaction costs.

Nonetheless, incorporating risk premiums into interest rates is also problematic. For example, higher interest rates increase the probability of loan default and—often —the discontinuation of a farm business. This results in the repossession of collateral, which is costly and highly disruptive to both individuals and communities. Furthermore, using interest rates to compensate lenders for high credit risk increases interest payments, reduces farm profitability and repayment capacities, and hampers investment in production-expanding technologies. In addition, the potential for crop failures increases credit risk, resulting in higher interest rates on agricultural operating, intermediate, and real-estate loans.

The availability and use of agricultural insurance reduces credit risk, lowers interest rates, improves repayment capacities, increases credit availability, and reduces financial and business risk. Crop insurance costs, however, can also be substantial. Crop insurance is subject to relatively high monitoring costs, requires large amounts of high-quality data to establish actuarially sound premium rates, and is inherently subject to moral hazard and adverse selection problems.

The pure risk premium component of interest rates in the absence of insurance is exactly equal to an insurance pure risk premium if the policies perfectly insure against loan default perils. As a result, insurance increases business costs only in the sense that it increases transaction costs. Insurance premiums are not separate, risk-related costs. In the absence of transaction costs, insurance premiums and the risk component incorporated into risk-adjusted interest rates would be identical. Risk is a cost of business activity, regardless of how it is managed.

## 1.2. Risk Management by Credit and Insurance Firms

Agricultural credit and insurance firms acquire risk through lending and insurance business practices. However, they must also balance the risk of loan defaults and insurance indemnities while maintaining adequate capital reserves. That is, above-average loan defaults or unexpectedly large indemnity payments require sufficient equity capital to maintain business integrity. As with any business firm, credit and insurance companies must have sufficient capital to manage unexpected cash outflows. Consequently, credit and insurance firms must decide how to manage this risk.

# 2.0. Agricultural Risk Management

Agriculture production is subject to many risks. For example, crop yields are highly influenced by unpredictable weather and climatic factors. In addition, crop and input prices vary with world, regional, and local factors that often cannot be accurately predicted. Investments in many technologies also have uncertain outcomes. Finally, changes in government domestic and trade policies can influence the profitability and repayment capacities of agricultural operations.

## 2.1. Agricultural Risks

Agricultural risks refer to events that negatively impact agricultural production or profitability. Risks can be characterized as:

1. Production risks, including extreme weather conditions, disease, pest infestations, and technology failures;
2. Market risks, including changes in output prices, market access, and input availability and prices;
3. Financial risks, including insufficient funds for debt repayment or family living expenses, a lack of credit availability, increased interest rates, and low returns from investments in new technologies;
4. Political and policy changes resulting in market distortions, unfavorable changes in exchange rates, and credit availability;
5. Legal risks, including ambiguous contract law, inconsistent judicial decisions and bankruptcy rulings, and other legal uncertainties;
6. Personal and human resource risks, including the deterioration of personal relationships and uncertainty regarding the health, safety, and lives of owners, managers, and employees.

## 2.2. Risk-Management Strategies

Managing agricultural (and other business) risks usually involves using more than a single approach. Risks can be avoided, absorbed, or transferred. The least sustainable management practices occur when risks are ignored.

**2.2.1. Risk Avoidance.** Avoiding risk is a common approach to risk management. That is, agricultural producers, financial entities, and other firms frequently make rational decisions that mitigate risk by deciding *a priori* to not incur it. For example, agricultural producers realize that their profession is relatively risky because of their use of machinery and the management of livestock. Hence, managers and employees often use a variety of safety measures when using machinery and during animal husbandry activities. In the case of machinery, shields and guards protect operators from moving parts, and operators often adopt safety practices such as turning off engines and setting parking brakes before dismounting. In the case of livestock husbandry, operators often use livestock restraining devices while administering animal health products. Other firms avoid risk by deciding against investing in various opportunities. Firms also institute a variety of safety policies to protect employees from injury and plan for management successors in the event of illness or death.

In terms of financial risks, agricultural producers and other firms often refrain from increasing debt commitments, purchasing untested technology, investing in high-risk investments, and aligning themselves with single-source input suppliers and output purchasers. In addition, producers often experiment with new seeds and chemicals before completely adopting either.

Although avoiding risks seems to be a reasonable approach to risk management, these strategies are not likely to maximize expected profits. That is, higher-risk enterprises are associated with the potential for higher returns on average. Consequently, the process of avoiding risk also produces lower average returns. Producers continually evaluate these risk/return trade-offs.

**2.2.2. Risk Absorption.** For some risks, agricultural producers and other business managers simply decide to absorb or accept the potential for adverse outcomes. However, business firms often employ a variety of strategies to self-insure against losses. For example, producers may engage in enterprise or regional diversification.

In terms of financial risk management, producers often retain excess cash reserves, debt-carrying capacities, and equity to obviate production and price risk. In many cases, producers augment farm income with off-farm income sources that diversify their income stream. Leasing or sharing of land, equipment, and livestock provides additional risk-management opportunities.

**2.2.3. Risk Transfer.** In addition to avoiding or absorbing risk, agricultural producers (and other businesses) often employ formal risk-transfer strategies as a means for managing risk. Formal risk-transfer mechanisms are not costless, however, as those who accept risk from others must be compensated. In terms of input and output prices, common mechanisms include forward contracting, futures contracts, and commodity options. Risk associated with life, health, and property is often transferred using formal insurance markets. Insurance products that transfer agricultural yield or revenue risks are widely available in developed economies.

Some risks are common across broad financial spectrums. For example, the recent global economic recession affected most economic sectors around the world. These types of risks are often referred to as "market" or "aggregate" risks. Aggregate risks are not diversifiable because they affect broad economic sectors. These risks are also commonly referred to as "systemic" or "systematic risk," but are probably best thought of as "nondiversifiable" risks.

Conversely, many risks are unique to a specific sector or region. Furthermore, even if risks are not unique—but simply uncorrelated with those occurring in other sectors or regions—then such risks are termed "unsystematic" or "diversifiable." Diversifiable risks are those that can be reduced by combining them in a portfolio, which provides the basis for value creation by financial, credit, and insurance companies. Developing portfolios that contain risks with low correlations among them is the basis of all insurance activity.

## 2.3. Agricultural Insurance

Agricultural insurance transfers insurable risk from a producer (the insured) to another party (the insurer) by means of a formal contract (an insurance policy). Insurance premiums are paid by the insured to an insurer to compensate the latter for the acceptance of risk and for other business costs.

Insurance policies are pooled to obtain diversification benefits and reduce average risk levels. Pooling often needs to occur across industries, regions, and time.

**2.3.1. Insurable Agricultural Risks.** Ideally, insurable risks are those that meet specific criteria:

1. The probability of an adverse event is calculable. A long time series of high-quality data is frequently needed to calculate probabilities;
2. The cause-and-effect relationships between risk and loss occurrence are identifiable;
3. The risk and size of an adverse event are beyond the control of the insured and the insurer;
4. The risk of an adverse event for one insured producer should not be perfectly correlated with the risk for others.

If any of these criteria are violated, additional strategies must be developed to account for these deficiencies. These criteria are seldom met, and insurance programs must account for these "imperfections."

**2.3.2. Noninsurable Agricultural Risks.** Often, one or more of the above criteria are not met by agricultural insurance situations. Such situations are then only insurable through reinsurance mechanisms with private companies or governments.

**2.3.2.1. Risks are not Calculable.** A lack of high quality data is the primary cause of some agricultural risks being uncalculable. Lengthy time series of high-quality yield and price data provide the best means for calculating future risk. Data are often unavailable for individual producers, especially in developing economies. Consequently, insurance rating must often resort to using parametric rather than empirical approaches. These approaches usually result in higher premium rates because of commensurately higher loading factors.

**2.3.2.2. Relationship between Losses and Cause of Loss.** Much of the process of underwriting involves identifying cause-and-effect relationships between an insured loss and a noninsured loss. For example, yields that fail to meet expectations could be caused by an insured peril such as hail or by an uninsured peril such as the use of poor-quality seed. Observationally, however, the results of these two events are equivalent and the monitoring costs of avoiding this problem are often quite large for agricultural insurance products.

**2.3.2.3. Moral Hazard.** Moral hazard refers to situations in which an insured can behave in a manner that influences the likelihood or size of an indemnity payment (i.e., a loss). The complexity, specificity, and high monitoring costs of production agriculture create considerable opportunity for moral hazard activity.

**2.3.2.4. Nonindependent, Systemic Risk.** The nature of agricultural risk often causes risks to be correlated across counterparties. That is, an adverse weather event often influences an entire region rather than a single producer. Such risks require pooling or diversification across regions and, often, across industries. If primary financial institutions retain risk, their capital reserve levels must be higher relative to less-correlated risks.

# 3.0. Risk Transfer and Reinsurance

Agricultural production and revenue risks are somewhat diversifiable across production sectors and regions. However, monitoring costs increase rapidly across both dimensions. In addition, crop yields within and across regions are often highly correlated. For example, widespread weather events (e.g., drought) usually affect many crops and production regions. The same can be true for crop diseases and pests. Additionally, agricultural commodity prices are often highly correlated with each other. However, some risks are uncorrelated with other regional and world economic sectors. Hence, primary agricultural insurers often cede risk to reinsurers who compile diversified risk portfolios. In addition, primary insurers often do not possess enough capital to provide indemnity payments if widespread yield losses do occur.

## 3.1. Reinsurance

Reinsurance refers to the transfer of risk from a primary insurer (i.e., an issuing agency) to another insurance company (reinsurer) or a government. This process allows insurers to develop risk portfolios that help manage overall risk. Individual issuing agencies often do not have the capacity to develop their own risk portfolios because they specialize in crop insurance. These specialized companies are often located in regions where their specific knowledge helps reduce transaction and monitoring costs but also results in limited opportunities for risk-portfolio diversification.

Primary insurers often transfer agricultural insurance portfolios to (usually) large reinsurers, which use these risks to diversify their own risk portfolios. These companies develop risk portfolios across sectors and countries while maintaining sufficient capital adequacy levels. Reinsurers charge fees to primary insurers for accepting and transferring risk.

Reinsurers are usually large, international companies that are well diversified across regions, countries, and economic sectors. Many governments also provide reinsurance opportunities. In many cases, governments provide stop-loss reinsurance services that support both primary insurers and reinsurers.

Various arrangements among primary insurers, reinsurers, and governments are used to share risk. For example, primary insurance companies commonly retain the first 5–10% of liability. In some cases, governments accept some or all of the liability for specific agricultural producers who would not normally be insurable by primary insurers. In the United States, for example, this may represent 5–15% of liability. Reinsurers accept (for a fee) 75–90% of the remaining indemnity payments. However, governments often provide stop-loss activities for liabilities that exceed 400% of premiums generated by agricultural insurance sales in any given year. Stop-loss programs protect primary insurers and reinsurers against widespread catastrophic losses.

## 3.2. Reinsurance Mechanisms

Primary insurers sell insurance contracts to agricultural producers. In exchange for insurance premiums (fees), the primary insurers agree to offset yield losses that exceed a specific trigger level. Essentially, insurance premiums represent the costs of transferring risk from a producer to the primary insurer. Over the long run, actuarially sound premium rates will cause the sum of indemnity payments plus transaction costs to equal

collected premiums. However, while premium collections may be stable over time, annual indemnity payments can vary greatly based on weather, disease, pests, and market events. As a result, primary insurers often transfer at least some of their indemnity exposure to reinsurers.

Multiple mechanisms exist for transferring risk from primary insurers to one or more reinsurers. Each involves differences in risk exposure and indemnity payment responsibility. In every case, primary insurers pay fees to reinsurers for accepting ceded risk.

## 3.3. Reinsurance Agreements

Reinsurance agreements provide the terms of risk transfer between insurance companies. Two general types of reinsurance agreements exist depending on the manner in which risk transfer occurs from an organizational and legal perspective: **treaty** and **facultative cover**.

**3.3.1. Treaty.** Treaty forms of reinsurance agreements obligate reinsurers to accept the entire liability stated in the reinsurance agreement. That is, the terms and conditions for the contract are specified and agreed to prior to the transfer of risks. These contracts are usually continuing agreements between a primary insurer and a reinsurer. Either party can terminate the arrangement at specified points in time, but the exact nature and quantity of risk transferred is subject to negotiation whenever a new set of risks are considered.

Treaty agreements have several advantages, including:

- Increased volume of reinsurance operations;
- Uniform distribution of risks among parties;
- Lower transaction costs;
- Longer-term relationships between parties;
- Added flexibility for reinsurers.

The main disadvantage of treaty agreements is that some risks may be outside the scope of the agreement or so large that they require large capital reserves. Additional agreements are often necessary in these cases.

**3.3.2. Facultative Cover.** Facultative cover agreements are noncontinuing arrangements in which parties negotiate risk transfer details whenever new liabilities are incurred. Facultative agreements have several advantages, including:

- Providing primary insurers with a larger set of reinsurance options, as they can negotiate with more than one reinsurer;
- Allowing specific risks to be transferred to reinsurers with expertise in specific areas.

Disadvantages of facultative agreements include:

- Higher transaction costs, as these agreements are usually complex and time consuming;
- Increased operational costs because of additional monitoring and reporting activities;
- Additional reinsurance submissions for multiple reinsurers;
- Increased uncertainty regarding long-term business relationships.

## 3.4. Forms of Reinsurance

Multiple forms of reinsurance exist, and each is defined by the manner in which risks are distributed between a primary insurer and a reinsurer (or reinsurers). In general, these approaches are classified as "proportional" or "nonproportional" agreements, although combinations of the two also exist.

Proportional reinsurance agreements distribute risk between a primary insurer and a reinsurer in a proportional arrangement. That proportion also determines each party's share of premiums as well as liabilities. Nonproportional reinsurance, in contrast, implies that indemnities are determined exclusively by loss amounts, and no proportionality between premiums and indemnities exists.

**3.4.1. Proportional Agreements.** A variety of terms are often used interchangeably to describe proportional reinsurance agreements, including Co-Pay, Quota Share, and Percentage Participation. Dollar One arrangements are usually co-pay arrangements in which the reinsurer shares indemnity payments without incurring a deductible.

All of these arrangements are similar in the sense that a reinsurer (or reinsurers) contractually agrees to accept a fixed share of insurance liability for specific risks that are transferred from a primary insurer. Reinsurance premiums paid by a primary insurer to a reinsurer are proportional to the share of liability in addition to other loads and fees. The same proportion applies to indemnity payments. In general, these arrangements start with the first monetary indemnity outlay (i.e., Dollar One).

Figure 1 illustrates an example in which a primary insurer transfers 50% of accepted risk to a reinsurer while retaining the remaining 50%. In addition, the primary insurer also transfers 50% of insurance premiums to the reinsurer, while the reinsurer is responsible for 50% of indemnity payments regardless of the size of losses.

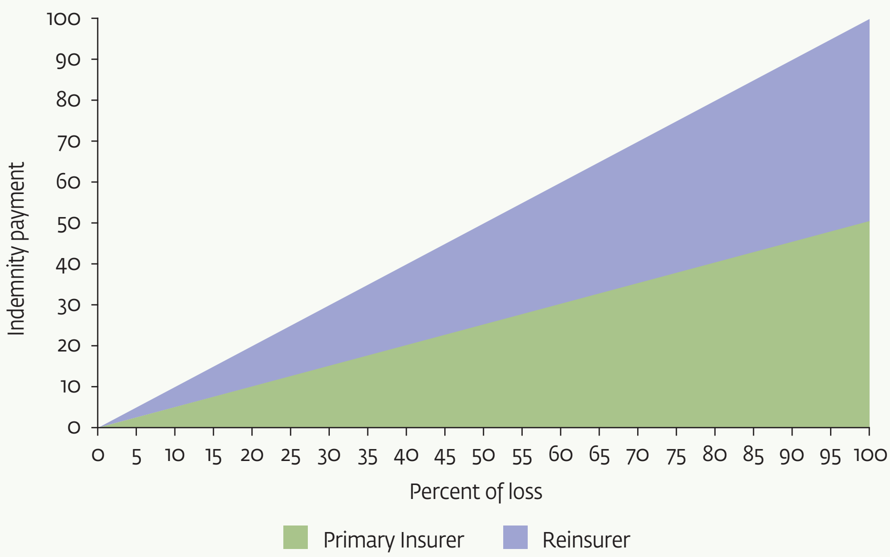


**Figure 1.** 50-50 Proportional Reinsurance Agreement

**3.4.2. Nonproportional Agreements.** Nonproportional reinsurance arrangements include a variety of options involving reinsurance deductibles. A deductible is often applied to a loss for which the primary insurer is responsible. Losses that exceed the deductible are then the responsibility of a reinsurer. Governments are frequently also responsible for indemnities through stop-loss or high-risk producer pools.

Nonproportional insurance agreements are generally used for two purposes. First, using deductibles provides incentives for primary insurers to make appropriate business decisions in terms of insurance sales and monitoring. Consequently, reinsurance loading fees are often smaller for nonproportional agreements. Deductibles are often between 5% and 10% of total liability or approximately equal to the pure risk premium. Second, private sector would likely not provide crop insurance products in cases where large, severe losses could potentially occur. So, the nonproportionality of government stop-loss or high-risk pools are often used to encourage insurance market development.

A variety of nonproportional reinsurance agreements exist, including:

1. Excess of Loss as a Cover per Risk, which provides protection for a portfolio against a particular risk;
2. Excess of Loss as a Cover per Catastrophic event, which provides protection against large losses;
3. Stop-Loss, which provides protection against losses accumulated over a certain period of time in which all losses above a specified level become the responsibility of a single party.
4. Layers or tranches.

The majority of nonproportional reinsurance agreements are layered or tranche arrangements, in which liability is defined by individual tranches or as percentage amounts. Tranches are hierarchical arrangements in which one entity is responsible for a specific amount of indemnity in a specific order. If losses exceed the amount specified in the first tranche, then a second entity is responsible for the next amount (tranche). In many cases, a primary insurer contracts with a reinsurer who then sells further tranches to others. As a result, many levels or tranches may exist for a single set of liabilities.

**3.4.2.1. A Fixed Amount Tranche Example.** Although nonproportional insurance programs often have unique layering aspects, the process of allocating liability among layers is similar across products. The following example illustrates common themes among these programs.

Table 1 presents a reinsurance program for a 700 million UAH liability portfolio that includes four participants – a primary insurer, a first reinsurer, a second reinsurer, and a government. In this example, the primary insurer retains the first 50 million UAH of liability, the first reinsurer is assigned the next 150 million UAH (the amount between 50 and 200 million UAH), and the second reinsurer accepts 100 million UAH (the amount between 200 and 300 million UAH). The government entity then provides a stop loss for liabilities that exceed 300 million UAH.

**Table 1.** Layered (Tranche) Reinsurance for a 700 Million UAH Liability.

| | | Indemnities Paid | | | |
|---|---|---|---|---|---|
| Scenario | Total Indemnities | Primary Insurer | First Reinsurer | Second Reinsurer | Government |
| A | 30 | 30 | | | |
| B | 50 | 50 | | | |
| C | 125 | 50 | 75 | | |
| D | 250 | 50 | 150 | 50 | |
| E | 500 | 50 | 150 | 100 | 200 |

If indemnities are less than or equal to 50 million UAH (i.e., Scenarios A and B in Table 1), the primary insurer provides the entire amount of indemnity payments. Scenario C presents outcomes for a case in which 125 million UAH of indemnities has been generated. In this case, the primary insurer is responsible for 50 million UAH and the first reinsurer pays the remaining 75 million UAH. In scenario D (indemnities totaling 250 million UAH), the primary insurer provides 50 million UAH of indemnity payments, the first reinsurer provides 150 million UAH, and the second reinsurer provides 50 million UAH.

**3.4.2.2. A Percentage Tranche Example.** Layered reinsurance may be defined either as fixed tranches (as presented in Table 1) or in terms of percent of loss, as illustrated in Figure 2. Percent of loss is calculated by dividing indemnities by total liability. In this case, a primary insurer is responsible for indemnities up to the first 10% of loss (a 10% deductible). After this point, the reinsurer is responsible for the Percent of Loss.

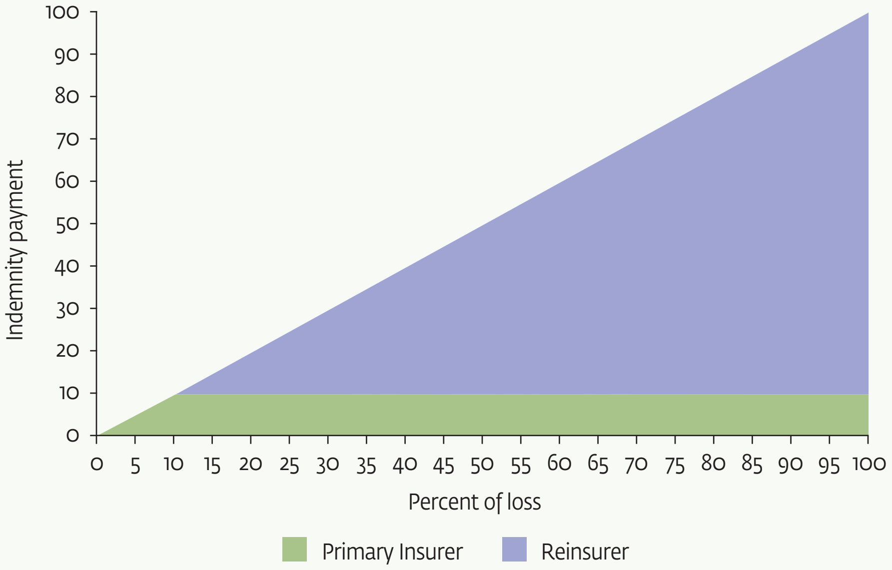


**Figure 2.** 10-90 Tranche Indemnity Retention for Primary Insurers and Reinsurers

**3.4.3. Combinations of Proportional and Nonproportional Reinsurance.** Reinsurance programs can also include elements of proportional and nonproportional liability assignments. Figure 3 illustrates a case in which 8.5% of liability is assigned to the primary insurer as a deductible. However, liabilities that exceed this deductible level are shared proportionally between the primary insurer and the reinsurer. Specifically, the primary insurer retains 15% of indemnity payments for liabilities exceeding 8.5% of the total, and the reinsurer accepts the remaining 85%.

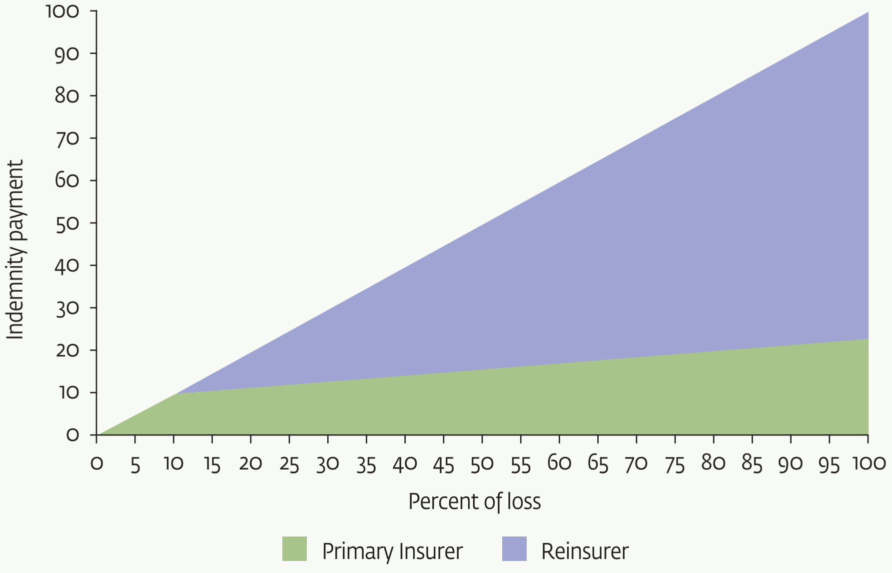


**Figure 3.** 8.5–91.5% Tranche/Co-Pay Indemnity Retention for Primary Insurers and Reinsurers

## 3.5. Ukrainian Example

Substantial differences exist in terms of sources of indemnity payments depending on the structure of reinsurance agreements. Consider the following Ukrainian example, for which annual yield data are obtained from 1987 through 2012 and trigger yields are set at 70% of the average yield. Figure 4 presents the yield data and trigger yield for each year. Indemnity responsibilities are determined by a proportional 50% co-pay arrangement between a primary insurer and a reinsurer. In each year for which an indemnity is triggered, the two entities share payment of indemnity amounts equally. Over the entire period, both the primary insurer and reinsurer payments average 4% of total liability per year.

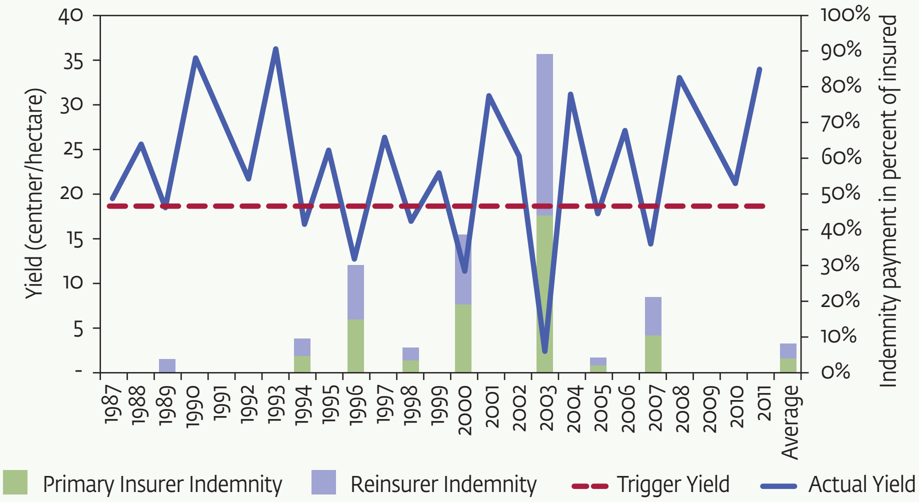


**Figure 4.** 50% Co-Pay Arrangement between Primary Insurer and a Reinsurer.

Figure 5 presents a nonproportional percentage arrangement in which the primary insurer is responsible for the first 10% of liability, which represents a 10% deductible. Beyond this amount, the reinsurer has a tranche that makes it responsible for the remaining 90% of liability. Over the entire period, the primary insurer's indemnity payments average 2% of total liability per year, while the reinsurer's indemnity payments average 6% per year.

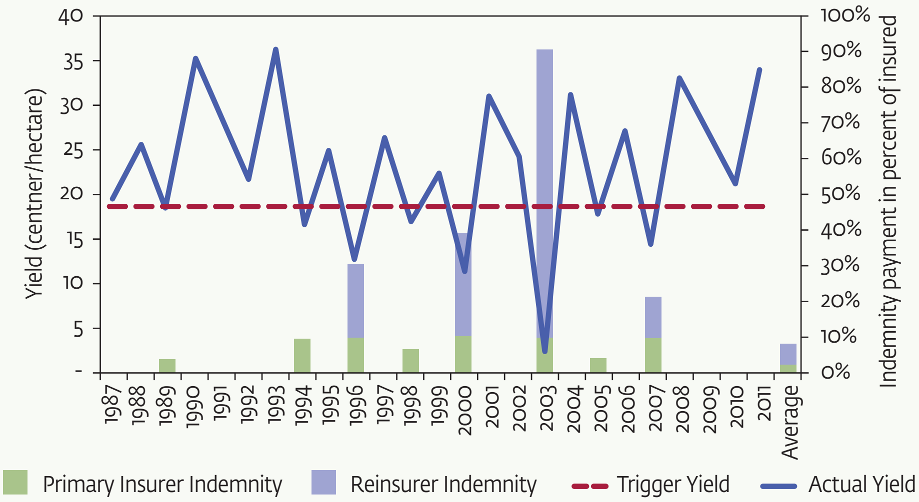


**Figure 5.** 10-90 Tranche Arrangement between Primary Insurer and a Reinsurer.

Figure 6 presents a third scenario. In this case, a tranche/co-pay arrangement makes the primary insurer responsible for the first 8.5% of liability, which represents a deductible. Beyond this amount, the primary insurer is proportionally responsible for 15% of the remaining liability and the reinsurer pays the remaining 85%. Over the entire period, the primary insurer's indemnity payments average 1% of total liability per year, while the reinsurer's indemnity payments average 7% per year.

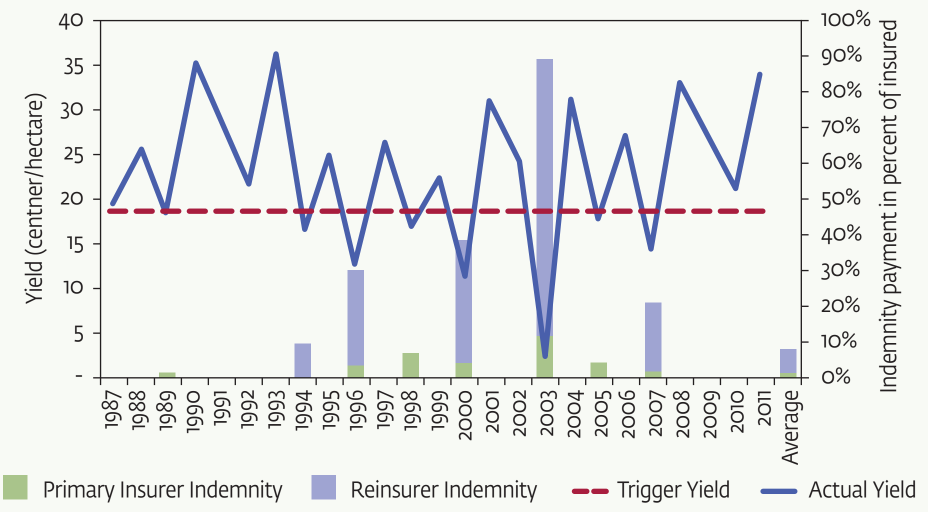


**Figure 6.** Tranche/Co-Pay Arrangement between Primary Insurer and a Reinsurer.

In summary, the primary insurer has larger indemnity responsibilities under co-pay arrangements in years of large losses.

## 3.6. Reinsurance Costs

The costs of obtaining reinsurance depend on pure risk premium rates as well as many other factors, including the number and depth of tranches, monitoring costs, and liability amounts. In addition, loads charged by reinsurers account for other elements such as servicing costs, program design, program integrity, underwriting issues, political, judicial and legal risks, personnel competency, reputation of involved parties, and the costs of capital. High loads are applied to countries without stable, equitable, and well-developed judicial systems. In addition, many of these situations are also coincident with poorly constructed contractual law and property rights. Reinsurance rating is discussed in the Actuarial Manual.

## 3.7. Document Package for Reinsurance

Primary insurers develop reinsurance submissions are as business proposals for consideration by reinsurers. Reinsurers use these submissions to evaluate their willingness to offer reinsurance and to determine appropriate risk transfers, costs, and loads. Some reinsurance companies have standardized forms for reinsurance submissions. Others simply provide a general outline. In all cases, reinsurers require similar information to make sound business judgments.

Reinsurance brokers are sometimes employed to facilitate a reinsurance submission. For example, the two largest reinsurance brokers—Aon Re Global and Guy Carpenter—collectively broker 60% to 70% of reinsurance contracts. Although reinsurance submissions often contain publically available data, each reinsurer uses proprietary methods for calculating reinsurance rates.

Reinsurance submissions generally comprise the following sections:

- Executive summary;
- Introduction;
- Actuarial documentation;
- Product administration;
- Underwriting;
- Loss adjustment.

**3.7.1. Executive Summary.** The executive summary provides information regarding

1. Value of the insurance product;
2. Competitive forces;
3. Specific form of the insurance product;
4. Description of key underwriting provisions.

Eventually, the rating portion of a reinsurance submission is submitted to a reinsurer's actuary experts, who evaluate the adequacy of suggested rates.

**3.7.2. Introduction.** The introductory section provides the background for the insurance product, including:

1. Relevance of the insurance plan;
2. Product need;
3. Product development strategies;
4. Field product design;
5. Examples of protection offered;
6. Insurance related parties and overall business model;
7. Background and market overview;
8. Situation and outlook for the crop being insured;
9. Pilot target market;
10. Forecast of premiums, liability, and market penetration as product is introduced and matures.

**3.7.3. Actuarial Documentation.** Actuarial documentation represents a highly detailed discussion of premium-rating procedures. The documentation must include:

1. Description and discussion of the data used to establish rates;
2. Description of the rating method;
3. Discussion of assumptions used in rate formulation;
4. Rating example;
5. Risk related rates and premiums;
6. Simulated book-of-business based on simulated loss history;
7. Estimated probability distribution of aggregate indemnity payment and reinsurance payment at the 50th, 20th, 5th and 1st percentile.

The primary insurance company must describe the data and procedures used to develop primary insurance rates. Providing an assessment of expected median indemnity payments in addition to expected average payments is also useful. Reinsurance companies use this information to evaluate the capital reserves needed for insurance products.

**3.7.4. Product Administration.** This section describes various record-keeping strategies, accounting processes, and operational administration for each product. The timing and processing of sales, marketing, education, data collection, and indemnity payments must be clearly presented.

**3.7.5. Underwriting.** Underwriting procedures are critical for successful reinsurance submissions. The technical details regarding policy provisions and farm-management practices must be clearly delineated. All schedules, events, and deadlines must be written to avoid ambiguity. In addition, detailed legal and policy issues must be developed.

**3.7.6. Loss Adjustment.** Reinsurance submissions must clearly explain loss-adjustment processes. All standards and protocols surrounding loss-adjustment procedures must be presented. The practical aspects of who, where, and how loss adjustment is to be undertaken is an important element of this section. Loss-adjustment data requirements, documentation, and compliance activities must be defined.

A major aspect of loss adjustment that directly affects reinsurance submissions involves a clear explanation of quality control with respect to loss adjusters. Loss adjusting may be undertaken by contractual arrangements or with salaried employees. Procedures must provide clear delineation between crop-insurance sales agents and loss adjusters because blurring these activities can be a source of insurance fraud.

## 3.8. Requirements for Data, Reporting, and Accounting

Reinsurance agreements require various data reporting and auditing procedures to be specified. These requirements ensure that reinsurance premiums are paid accurately and clarify reinsurance indemnity obligations. Total liabilities must be defined in reinsurance agreements.

Reporting schedules must be delineated for several activities, including:

- Expected total liability prior to reinsurance attachment;
- Total liability incurred immediately after sales closing dates;
- Forecasts of indemnity payments prior to due dates;
- Crop and growing-condition reports and indemnity forecasts throughout the growing season;
- Final indemnity reports due shortly after growing season and reinsurance payment notification.

Accounting requirements must allow reinsurers to audit transactions to verify liability and indemnity payments.

## 3.9. Participation

The level of producer participation in any given crop-insurance product is a major concern for reinsurers. Reinsurers desire a sufficient volume of participation to spread transaction costs. Minimum target product size is approximately $200 million of total liability. For pilot projects, the target liability is often between $5 million and $20 million.

Producer participation in any crop insurance product is influenced by:

- Risk-management needs;
- Producer awareness and education;
- Producer expectations of indemnity payments relative to insurance-premium costs;
- Producer levels of risk aversion;
- Relationship of crop-insurance products to credit availability, legal requirements, and cross compliance with other government programs;
- Producer confidence in program integrity.

Reinsurers are also concerned about "adverse selection," which occurs when high-risk producers in an area realize that an insurance premium has been misrated for their particular situation. That is, some producers face rates that are too high for their operations, while others face rates that are too low. In addition, producers are likely to have more knowledge of their own production risks than insurers. Higher-risk farmers may therefore exploit this knowledge and disproportionally enroll in underrated insurance products. This situation can result in participation erosion if low-risk producers decide not to participate and high-risk producers decide to disproportionally purchase a product.

Assume that a crop-insurance premium is initially based on expected losses across all farmers and that one-half of the highest-risk producers purchase the product. Indemnity payments will be higher than expected, and premium rates will eventually be adjusted upward to match this outcome. Higher rates will discourage more low-risk producers from participating because their premiums are now higher than their expected losses. Consequently, these producers will reduce their participation. If the process continues, fewer and fewer producers will purchase the insurance product.

Various actions can be taken to mitigate participation erosion, including:

- Risk pooling, which separates producers with similarly high- or low-risks into separate pools that allow for appropriate rating based on risk exposure;
- Subsidizing premiums so that an insurance product is desirable to both high- and low-risk producers;
- Providing products that are attractive to those with high levels of risk aversion;
- Requiring participation in exchange for receiving other types of government support;
- Developing proxy or area insurance products so that all farmers have equal likelihood of receiving indemnities which reduces adverse selection;
- Engendering producer confidence in product and program design.

Finally, reinsurance companies are also attracted to insurance products that are standardized across areas and countries. These products allow for common program administration, loss adjustment, and accounting.

# 4.0. Risk Absorption and Capital Adequacy

The safety and soundness of financial institutions has become a leading issue because of the recent global financial crisis. Historically, financial crises have occurred approximately every twenty years. The worst financial crisis in the last seventy-five years occurred in 2008–2009. Financial crises have a variety of causes and are manifest in various ways. The impact on individuals ranges from minimal to severe depending on whether or not economic recessions result in high unemployment, loss of equity, or reductions in standards of living. Avoiding future financial crises has become a national and international priority, and financial institutions also have vested interests in avoiding financial crises.

The trade-off between risks and rewards generates difficult decisions about whether to take conservative versus aggressive financial management strategies. Firms must decide between sacrificing long-term economic growth versus financial risks. Efforts to minimize the impact of financial risks (i.e., the elimination of all risk) are unacceptable because these strategies diminish the potential for long-term growth.

## 4.1. Financial Institution Regulation

The financial sector is highly concentrated in most countries. For example, in the United States, about 80% of all bank assets are owned by the seven largest banks. Bank-related problems are usually contagious due to financially connected counterparty relationships. As a result, financial difficulties in a single, large banking institution can affect other financial institutions. In addition, many factors affect lending institutions similarly. For example, increased unemployment or reductions in home prices are likely to generate home mortgage defaults across all lenders. Consequently, many governments regulate financial institutions, including banks, insurance, securities, thrift/credit unions, and futures markets. Some countries also regulate secondary financial markets and associated service industries, such as accounting and auditing firms.

The goals of regulation (GAO-09-216 "Financial Regulation") are to:

1. **Ensure adequate consumer protections.** The profit maximizing goals of financial institutions often encourage the sale of unsuitable or fraudulent financial products and participation in unfair or deceptive acts or practices. Government regulators attempt to address informational disadvantages that consumers and investors may face by reporting information about products and services and monitoring business conduct and sales practices.

2. **Ensure integrity and the fairness of markets.** Because some market participants may manipulate highly concentrated markets to obtain unfair gains in ways that are not easily detectable by other participants, regulators often establish rules and monitor market behavior. Their actions help prevent fraud and market manipulation, limit asset-pricing problems, and encourage efficient market activity.

3. **Monitor the financial safety and soundness of institutions.** Market opportunities can sometimes lead financial institutions to acquire excessive risks, which can have significant negative impacts on consumers, investors, and taxpayers. Regulators oversee risk-taking activities that promote the safety and soundness of financial institutions.

**4. Ensure the stability of the overall financial system.** Because shocks to a single financial sector or institution can destabilize entire financial systems, regulators attempt to reduce systemic risk in various ways. For example, governments often provide funding (liquidity) to troubled financial institutions to avoid financial system contagion.

Most financial regulation has historically focused on banking institutions, but regulation has recently spread to other financial entities. Although the focus of this manual is on insurance entities, understanding of banking regulations provides key insights into the regulation of insurance providers. Insurance regulation is neither as standardized nor as widespread as banking regulation, but the evolution of insurance regulation is guided by banking regulation. Given the inherent linkages between credit and insurance (see the Credit and Insurance Manual), understanding banking regulation is instructive when considering insurance regulation. Insurance practitioners should have an intuitive understanding of banking regulation procedures. The following overview of Basel Agreement banking regulation is intended to provide an introduction to capital-adequacy principles.

## 4.2. Basel Agreements

Although banks and other financial institutions have been regulated to varying degrees for centuries, the modern era of bank regulation was initiated with the Basel Agreements, which originated in 1988 with the Basel Committee on Banking Supervision. The original Basel Agreements have been refined and modified. Although some nations have not fully adopted Basel principles, the Basel Agreements have become the guideline for worldwide bank regulation.

Basel I and Basel II provide the current guiding requirements. Basel III is scheduled for full implementation in 2019. Current Basel requirements are structured around three pillars:

**Pillar 1:** The measurement of credit, market, and operational risk;

**Pillar 2:** Capital adequacy;

**Pillar 3:** Market discipline through enhanced public disclosure or transparency.

Pillars I and II are referred to as the safety and soundness requirements. These requirements are designed to protect “fixed amount creditors” from the failure and insolvency of a financial institution and resulting financial system instability. Fixed amount creditors include bank depositors and potential insurance beneficiaries. The term “fixed amount” indicates that creditor payouts are not dependent on market outcomes. Consequently, the primary regulatory mechanism involves capital adequacy.

**4.2.1. Capital Adequacy.** Capital adequacy refers to financial institutions maintaining specific levels of equity capital as a buffer against adverse outcomes. This buffer allows these institutions to meet the obligations of fixed amount creditors. The required minimum amount of capital depends on the quantity and type of assets held by the institution and is calculated as:

$$\text{Required Capital} = 8\% \times \text{Risk Weight} \times \text{Asset Value}$$

The 8% of risk-weighted asset values is a regulatory requirement. Risk weights vary by asset type, as shown in Table 2. Asset values are obtained from an institution’s balance sheet. Off-balance-sheet items are also included in the Basel analysis. Table 2 also presents the capital required per dollar of asset value calculated as 8% of the risk weight.

Total required capital is the sum of required capital across all balance sheet assets, including off-balance items. Some claims on assets often receive special consideration, but such considerations are not consistent across countries. Agricultural real-estate loans exhibit similar risk to residential housing loans and so are often treated accordingly.

The Basel regulations not only address the quantity of required capital but also its quality. The regulations place particular emphasis on Tier 1 capital, common stock equity, and retained earnings. The regulations restrict the amount of Tier 2 capital that can be used to meet capital requirements. Tier 2 capital is considered to be of lower quality and includes most preferred stock and other capital that is not easily available for absorbing risk.

**Table 2.** Risk Weights and Capital per Dollar of Asset Value

| Claims on or secured by | Rating | Risk Weight(%) | Capital Per Dollar of Asset Value |
|---|---|---|---|
| Sovereigns | AAA/AA- | 0 | 0 |
| | A+/A- | 20 | 0,016 |
| | BBB+/BBB- | 50 | 0,04 |
| | BB+/B- | 100 | 0,08 |
| | Below B- | 150 | 0,12 |
| | Unrated | 100 | 0,08 |
| Banks and Securities | AAA/AA- | 20 | 0,016 |
| | A+/A- | 50 | 0,04 |
| | BBB+/BBB- | 100 | 0,08 |
| | BB+/B- | 100 | 0,08 |
| | Below B- | 150 | 0,12 |
| | Unrated | 100 | 0,08 |
| Corporates | AAA/AA- | 20 | 0,016 |
| | A+/A- | 50 | 0,04 |
| | BBB+/B- | 100 | 0,08 |
| | Below B- | 150 | 0,12 |
| | Unrated | 100 | 0,08 |
| Retail Products | | 75 | 0,06 |
| Residential Property | | 35 | 0,028 |
| Commercial Real Estate | | 100 | 0,08 |
| Cash | | 0 | 0 |
| Other Assets | | 100 | 0,08 |
| Overdue Loans | | Various Requirements | |
| Note: The weights are usually minimum values. | | | |

The Basel approach to risk-weighted capital requirements is commonly used by banks but is less commonly used by insurance companies. For banks, loans are assets and deposits are liabilities. For insurance firms, the cash generated by insurance premiums is an asset and potential indemnity payments are potential liabilities. The value of loans is more deterministic than the potential for indemnity payments. Furthermore, the frequency of crop insurance indemnity payments is usually much higher than the probability of loan defaults.

Basel III was scheduled to be implemented in 2013 but has been partially delayed until 2019. It places more emphasis on liquidity, capital quality, counterparty risk, cycle-based buffers, and a variety of other detailed issues. Additional modifications to Basel III requirements will probably occur prior to its full implementation.

Basel I and II have been widely criticized as being unduly burdensome (particularly on small banks) and blamed for increasing bank concentration. Some argue that increased concentration has caused a less stable financial system. From a financial perspective, a major shortcoming of Basel I and II is that they provide little consideration of diversification and correlated risks. In the United States, some of the Basel III requirements conflict with recently passed Dodd-Frank legislation. Similar internal regulatory conflicts are likely to occur in other countries as well.

**4.2.2. Value-at-Risk.** The Basel Agreements recognize Internal Risk-Based modeling approaches to a limited extent. Internal Risk-Based approaches, in general, encompass Value-at-Risk (VaR) methods. These approaches involve estimating asset values that could be lost in extreme or low-probability situations. The question posed is: What is the loss in value that can only be exceeded by a predetermined probability?

For example, assume that a portfolio has a value of $1 million and at the end of a predetermined period there is a 0.2% probability that the portfolio will have a value of $600,000 or less. The $VaR_{0.2\%}$ would be $400,000 in this case.

The terms "economic capital" and "capital-at-risk" are often synonymous with VaR. Furthermore, the notation $VaR_{\%}$ may use the % value as either the probability that a loss is equal to or greater than the stated percentage (e.g., 0.2%) or it may denote the loss equal to or less than 1 minus that percentage (e.g., 99.8%). The high or low value of the % value, which is referred to as the confidence level, CL, usually affords an obvious interpretation.

Figure 7 illustrates the VaR concept in a banking or insurance context. The expected loss, E[L], or the expected indemnity, E[I], is the amount imbedded in the interest rate for the risk of receiving less than a total loan payment or the pure risk insurance premium. The VaR is the minimum amount of capital required to offset unusually high loan losses or indemnity payments (defined as loan losses or indemnities exceeding expectations). UL and UI are unexpected losses or unexpected indemnities and are equal to the standard deviation of losses or indemnities. The variance of losses is denoted as V[L] and the variance of indemnities is denoted as V[I]. The standard deviations in both cases are the square roots of each. The use of the term "unexpected" is a misnomer because it is easily misinterpreted as a measure similar to VaR. The loss or indemnity curve is recognized as the probability density function of losses or indemnities. Required capital is directly related to a CL. The total area under the curve presented in figure 7 sums to 1, and the area under the curve to the left of the required capital level is the CL (i.e., 99.8%).

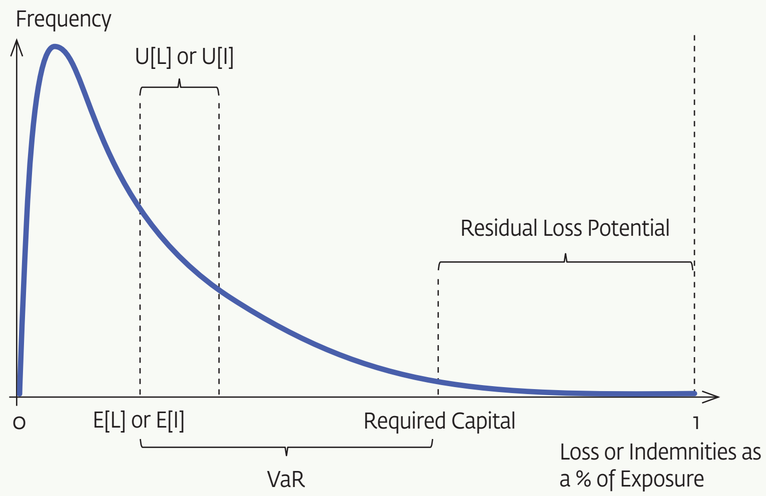


**Figure 7.** Portfolio Loss or Indemnity Distribution

The VaR approach uses statistical analyses specifically applicable to the portfolio of a financial institution. VaR is used to determine the level of required capital that is sufficient with confidence level CL to be institutionally safe and sound. When considering banking institutions, VaR focuses on loan portfolios, but in the case of insurance companies VaR is specific to insurance contract portfolios. The required capital is the sum of E[L] and VaR or the sum of E[I] and VaR.

The use of VaR in banking and insurance is similar from interpretive, analytic, and computational perspectives. The following discussion will move between banking/credit and insurance terminology. The terms probability of default (PD) in banking will be used interchangeably with frequency (Freq) in insurance as will loss-given-default (LGD) with severity (Sev), loss (L) with indemnity payment (I), and expected loss (E[L]) with expected indemnity(E[I]). Exposure refers to total liability or total loan volume. These terms are further discussed in the "Actuarial Methods" and "Credit and Insurance" manuals. Losses and indemnities are often expressed as a percentage of exposure, as presented in Figure 7.

In general, two approaches are used to calculate VaR for a given time horizon and probability: parametric procedures and empirical Monte Carlo simulations. The time horizon is the period for which the analysis is conducted. For example, the time horizon may be one year, which means that, say, $400,000 is adequate capital at a confidence level of 99.8%. Probability is often referred to as a confidence level particularly if it is specified

as losses or indemnities. The time horizon is a matter of convenience or determined by specific situations. For securities, the time horizon may be as short as a day. For banking situations, it is usually a year. In the case of crop insurance, the time horizon generally reflects a crop season.

**4.2.3. Parametric Value-at-Risk Approach.** The following parametric VaR approach generally follows that described in Bluhm, Overbeck, and Wagner (Introduction to Credit and Risk Modeling, Second ed., Chapman & Hall, CRC Press, 2010). Bluhm, Overbeck, and Wagner (BOW) contain substantially more detail at a mathematically sophisticated level. Nonetheless, a variety of parametric approaches can be used. The following is an example of one of these approaches.

The steps in the parametric approach are:

1. Compute the individual loan or insurance contract statistics, $E[L]$ or $E[I]$ and $UL$ or $UI$;
2. Convert the above statistics to a portfolio level while recognizing correlations between loan losses or indemnity payments;
3. Convert portfolio statistics into a probability density function;
4. Calculate VaR and required capital.

**4.2.3.1. Individual Statistics.** During the actuarial rating process, $E[L]$ or $E[I]$ must be calculated. Recall that

$$E[L]=(PD)(LGD) \text{ and}$$

$$E[I]=(Freq)(Sev)$$

Developing $UL$ and $UI$ is more complicated. The following discussion will focus on $UL$, but these processes are also relevant for calculating $UI$ in insurance situations. Let $L_D$ be a Bernoulli variable where $L_D$ = 1 when a loan defaults and $L_D$=0 when a full loan payment is made. Therefore, $E[L_D]=PD$. The variance of $L$ (loan loss) is

$$V[L]=V[(LGD)(L_D)].$$

Recall that $V[x]=E[x^2]-E[x]^2$, so

$$V[L]=E[LGD^2L_D^2]-E[(LGD)(L_D)]^2.$$

Two special cases are discussed. If $LGD$ and $L_D$ are independent, then

$$V[L]=E[LGD^2]E[L_D^2]-E[LGD]^2E[L_D]^2.$$

However, because $L_D$ is Bernoulli $L_D^2=L_D$ and $E[L_D]=PD$, then

$$V[L]=E[LGD^2]PD-E[LGD]^2PD^2.$$

Furthermore,

$$V[LGD]=E[LGD^2]-E[LGD]^2 \text{ so } E[LGD^2]=V[LGD]+E[LGD]^2,$$

which can be substituted into

$$V[L]=(V[LGD]+E[LGD]^2)PD-E[LGD]^2PD^2= V[LGD]PD+E[LGD)]^2PD(1-PD).$$

In an insurance context, the analogue is

$$V[I]=V[Sev]Freq+E[Sev]^2\,Freq(1-Freq),$$

where frequency and severity are assumed to be independent.

The assumption of independence is questionable. To illustrate, let *y* represent yield and *t* represent the insurance yield trigger. Then, for the normal distribution,

$$\begin{aligned} &E[I]=V[y]g[t]+(t-E[y]])G[t] \\ &Freq=G[t] \\ &Sev=\frac{V[y]g[t]+(t-E[y])G[t]}{G[t]}=\frac{V[y]g[t]}{G[t]}+(t-E[y]), \end{aligned}$$

where *g* and G are the normal pdf and cdf. Figure 8 illustrates the tradeoff between frequency and severity for an insurance example in which *E*[*y*] = 1 and *V*[*y*] = $0.28^2$ with various triggers ranging from 0.50 to 0.85. Obviously, the frequency and severity are positively correlated in this case, which violates the independence assumption.

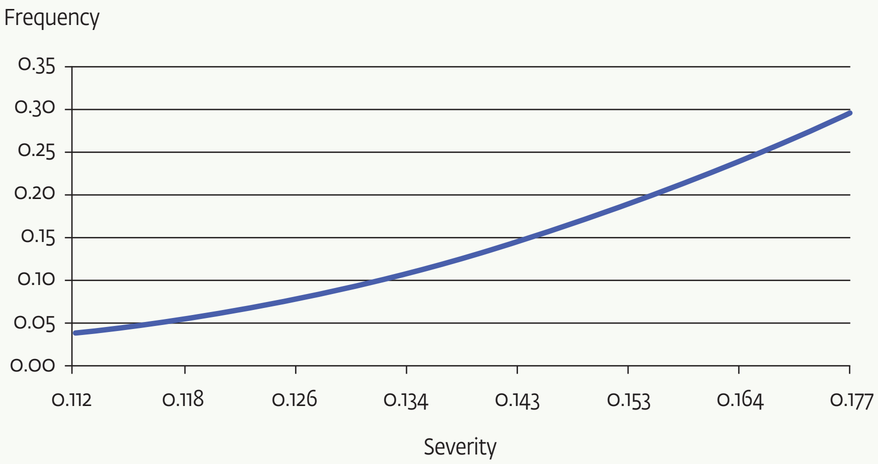


**Figure 8.** Relationship between Frequency and Severity

Alternatively, consider a distribution for which frequency and severity are unrelated (e.g., the Laplace). The Laplace distribution has a pdf and cdf given by:

$$h[y]=\frac{e^{\frac{y-E[y]}{b}}}{2b}$$

$$H[y]=\frac{e^{\frac{y-E[y]}{b}}}{2}$$

when *y* < *E*[*y*] and where $b=\frac{s_y}{\sqrt{2}}$. Therefore, *H*[*t*] is the frequency and

$$E[I]=\int_{-\infty}^{t}(t-y)h[y]dy=tH[y]-\int_{-\infty}^{t}yh[y]dy.$$

Integration by parts yields

$$\int_{-\infty}^{t} yh[y]dy = tH[t] - \int_{-\infty}^{t} H[y]dy = tH[t] - bH[t] = (t-b)H[t],$$

such that

$$E[I] = bH[t]$$

$$Sev = \frac{E[I]}{Freq} = \frac{bH[t]}{H[t]} = b.$$

Severity is a constant (*b*) which indicates that it is independent of frequency. Therefore, if *y* is distributed as a Laplace, then

$$V[I] = V[Sev]Freq + E[Sev]^2 Freq(1 - Freq)$$

holds, since frequency and severity are independent and the square root of *V*[*L*] is *UL*.

Given that *I* is the indemnity payment (i.e., *I* = *t* - *y*) and *f* [*y*] and *F*[*y*] are the pdf and cdf, the variance of *I* is

$$\begin{aligned} V[I] &= \int_0^t (I - E[I])^2 f[y]dy + \int_t^{\infty} (-E[I]]^2 f[y]\text{dy} \\ &= \int_0^t (I^2 - 2IE[I] + E[I]^2) f(y)dy + E[I]^2(1 - F[t]) \\ &= \int_0^t I^2 f[y]dy - 2E[I]\int_0^t If(y)dy + E[I]^2 F(t) + E[I]^2(1 - F(t)) \end{aligned}$$

Substituting *t* - *y* for *I* yields

$$\begin{aligned} V[I] &= \int_0^t (t-y)^2 f[y]dy - 2E[I]\int_0^t (t-y)f[y]dy + E[I]^2 \\ &= \int_0^t (t^2 - 2yt)f[y]dy + \int_0^t \text{y}^2 f[\text{y}]dy - 2E[I]^2 + E[I]^2 \\ &= \int_0^t (2t^2 - 2yt - t^2)f[y]dy + \int_0^t y^2 f[y]dy - E[\text{I}]^2 \\ &= 2t\int_0^t (t-y)f[y]dy - \int_0^t t^2 f[y]dy + \int_0^t y^2 f[y]dy - E[I]^2 \\ &= 2tE[I] - t^2 F[t] + \int_0^t y^2 f[y]dy - E[I]^2 \end{aligned}$$

The value of the integral containing $y^2$ is dependent on the distribution of *y*.

The second special case assumes that the yield variable, *y*, is normally distributed. Using primes to denote derivatives,

$$g'[y] = -\left(\frac{y - E[y]}{s^2}\right)g[y] \text{ so } yg[y] = E[y]g[y] - s^2 g'[y]$$

$$g''[y] = -\frac{g[y]}{s^2} + \left(\frac{y - E[y]}{s^2}\right)^2 \text{g[y]} \quad \text{so} \quad y^2 g[y] = s^4 g''[y] + s^2 g[y] + 2yE[y]g[y] - E[\text{y}]^2 g[y]$$

Therefore,

$$\begin{aligned} \int_o^t y^2 g[y]dy &= \int_o^t s^4 g''[y]dy + \int_0^t s^2 g[y]dy + \int_0^t 2E[y]yg[y]dy - \int_0^t E[y]g[y]dy \\ &= s^4 g'[\text{t}] + \text{s}^2 g[y] + 2E[y]\int_0^t yg[y]dy - E[y]G[t] \end{aligned}$$

Substituting for $g'[t]$ and $yg[y]$ using the above equations results in

$$\int_0^t y^2 g[y]dy = -s^4\left(\frac{y-E[y]}{s^2}\right)g[t] + s^2G[t] + 2E[y]\int_0^t (E[y]g[y] - s^2 g'[\mathrm{y}])dy - E[y]^2 G[t]$$
$$= (\mathrm{s}^2 + E[y]^2)G[t] - s^2(t + E[y])g[t]$$

Substituting into *V*[*I*] and simplifying yields

$$V[I] = 2tE[I] - E[\mathrm{I}]^2 + (s^2 + E[y]^2 - t^2)G[t] - s^2(t + E[y])g[t].$$

The following relationship can be developed for E[I]:

$$E[I] = \int_{-\infty}^{t} I[y]g[y]dy = V[y]g[t] + (t - E[y])G[t],$$

where *g*[*t*] and G[*t*] are the pdf and cdf, respectively, of the normal distribution. A similar result is shown in the Actuarial Manual.

**4.2.3.2. Portfolio Statistics.** The next step involves calculating statistics related to a portfolio. Let each individual loan or counterparty be denoted as *i* with *m* loans and a loss associated with loan *i* as $L_i$. The portfolio loss is

$$L_P = \sum_{i=1}^{m} L_i$$

and the expected portfolio loss is

$$E[L_P] = \sum_{i=1}^{m} E[L_i].$$

The general formula for the variance of the portfolio loss is

$$V[L_p] = \sum_{i=1}^{m}\sum_{j=1}^{m} Cov[L_i, L_j] = \sum_{i=1}^{m}\sum_{j=1}^{m} \sqrt{V[L_i]V[L_j]}\rho_{ij},$$

where *Cov* is the covariance operator and $\rho$ is the loss correlation. In some situations, it is useful to rewrite $V[L_P]$ as

$$V[L_P] = \sum_{i=1}^{m}\sum_{j=1}^{m} Cov[LGD_i L_{D_i}, LGD_j L_{D_j}].$$

If *LGD* is deterministic, then

$$V[L_P] = \sum_{i=1}^{m}\sum_{j=1}^{m} LGD_i LGD_j \sqrt{V[L_{D_i}]V[L_{D_j}]}\rho_{ij}.$$

The variance of the Bernoulli variable, $L_D$, is $V[L_D] = E[(L_D - PD)^2] = PD(1-PD)^2 + (1-PD)(-PD)^2 = PD(1-PD)$, so that

$$V[L_P] = \sum_{i=1}^{m}\sum_{j=1}^{m} LGD_i LGD_j \sqrt{PD_i(1-PD_i)PD_j(1-PD_j)}\rho_{ij}.$$

It is not uncommon for the above formula to be used to calculate $V[L_P]$. However, a more general formula is preferred and usually more accurate. If the loans or counterparties are homogeneous in *PD* and *LGD* and $\rho_{ij} = \rho$ if $i \neq j$ and $\rho_{ij} = 1$ if $i = j$, then

$$V[L_P] = LGD^2 PD(1-PD)m(1+(m-1)\rho)$$

or in the more general case

$$V[L_p] = \sum_{i=1}^{m}\sum_{j=1}^{m} Cov[L_i, L_j] = \sum_{i=1}^{m}\sum_{j=1}^{m} \sqrt{V[L_i]V[L_j]}\rho_{ij} = V[L]m(1+(m-1)\rho).$$

The insurance analog for the above equation is

$$V[I_P] = Sev^2 Freq(1-Freq)m(1+(m-1)\rho) = V[I]m(1+(m-1)p).$$

If the portfolio statistics are specified per dollar of exposure (i.e., where *L* or *I* has been rescaled by dividing by individual exposure) and if the counterparties are homogeneous, then as the number of counterparties become large $(m \to \text{infinity})$, the above statistics for lending become

$$E[L_P] = E[L_i]$$
$$V[L_P] = V[L_i]\rho$$

and for insurance become

$$E[I_P] = E[I]$$
$$V[I_P] = V[I]\rho.$$

The above statistics indicate that correlation plays a major role in the portfolio variance and is therefore a primary element of the level of VaR. However, determining the correlation is challenging. The obvious approach is to calculate the correlation of losses or indemnities from existing data. But even if such data are available, they often have been collected in an inconsistent and/or unrepresentative manner. For example, if underwriting standards on insurance policies or loans have changed, using indemnity payments or loan defaults may cause inaccurate calculations of correlations. Furthermore, loan loss or indemnity payment data will contain many zeros and only a few positive values. This causes the data to be particularly susceptible to sampling error. However, if a large volume of consistent data is available, then calculating the correlation directly from loss or indemnity payment data is the preferred approach.

An alternative to calculating loss or indemnity correlations is to use asset value correlations, which assumes that indemnity payments are paid when asset values decline. For loans, the logical thought process is that the value of assets decline as income declines and lower incomes lead to loan defaults. For crop insurance, the link between the asset insured (e.g., crop yield) and "default" is more direct because low crop yields trigger indemnity payments. The degree to which asset value correlations are aligned with loss or indemnity correlations varies depending on the characteristics of the probability distribution and the degree to which loss/default is caused or conjoined with low asset values. Consistent asset-value data are usually much more available than loss or indemnity data. If a lending institution or insurance company is large and well diversified with respect to other economic sectors, the correlation may be derived from publically available Beta coefficients.

Individual expected values and variances are usually calculated from actual losses or indemnity payments, while correlations are often calculated from underlying asset values. Correlation calculations require that both variables have the same numbers of observations. An additional implicit assumption is that the variables have some consistency between them. Whichever method or data is used to calculate individual expected values and variances, the data must be representative of actual expected values or variances. For example, a loss variance of one counterparty could be based on five years of data and the loss variance of another counterparty could be based on eight years (which could differ from that of the first counterparty) as long as the results were deemed to be a sufficiently accurate representation of the actual variances. Because of the different time periods considered, however, a correlation could not be calculated between the two sets of data. Asset value correlations are widely used as proxies for loss or indemnity correlations because asset value data are much more widely available and consistently gathered.

Proponents of asset correlations as a proxy for loss or indemnity correlations argue that loan defaults are highly correlated with lower asset values. This is clearly the case for crop insurance because low asset values (yields) trigger crop insurance indemnities. Furthermore, when indemnities or losses are associated with lower asset values, then the loss or indemnity variable will be ranked consistently with asset values. Thus, the rank correlation is the same between asset values and loss or indemnities. To the extent that rank correlation is representative of the usual product moment correlation, asset values correlations are a reasonable estimate of loss or indemnity correlations.

To illustrate the relationship between asset and indemnity correlations, consider a situation in which the asset value outcome is either 0 or 2. If the outcome is 0, an indemnity of 1 is paid. If the outcome is 2, the indemnity is not paid. Regardless of the correlation between the asset values of the two counterparties, the indemnity correlation will be exactly the same as the asset value correlation. Conversely, if the asset values are normally distributed, then the asset value correlations will not be the same as the indemnity correlations.

In the special case where severity equals 1 and is deterministic such that V[I]=Freq(1-Freq), it can be shown that (see Asset Correlation, Realized Default Correlation, and Portfolio Credit Risk, Jing Zhang, Fanlin Zhu, and Joseph Lee, Moody's/K.M.V., March 3, 2008, modified for an insurance setting)

$$\rho = \frac{G_2[G^{-1}[Freq], G^{-1}[Feq], \delta] - Freq^2}{Freq[1 - Freq]},$$

where

$\rho$ = indemnity or frequency correlation

$G_2$ =bivariate standard normal cumulative distribution

$G^{-1}$ = inverse standard normal cumulative distribution

$\delta$ = asset value correlation

If indemnity frequencies are available—including the frequency of any two counterparties incurring an indemnity (assumed constant across all counterparty combinations) —then (see Zhang, Zhu, and Lee)

$$\rho = \frac{JFreq - Freq^2}{Freq(1 - Freq)},$$

where *JFreq* is the joint probability that both counterparties incur an indemnity.

Under the Basal II IRB framework, economic capital is (see International Convergence on Capital Measurement and Capital Standards, Basel Committee on Banking Supervision, June 2006),

$Capital = Sev * G\left[G^{-1}[Freq] * \sqrt{1/(1-\delta)} + G^{-1}[CL] * \sqrt{\delta/(1-\delta)}\right] - Freq * Sev$ where *CL* is the confidence level. If *Freq* = 0.16, *Sev* = 0.21, $\delta$ = 0.28, and *CL* = 0.999, then the economic capital is 0.129 and total required capital is 0.163 (economic capital + EI where EI=Freq*Sev.)

Often, data are not in the form needed to develop desired correlations, but these correlations can sometimes be developed using alternative approaches. For example, assume that correlations among individual farm yields are

desired within a particular region for developing $V[I_p]$. Furthermore, assume that historical regional yield data are available along with either farm yield data or insurance rates for given coverage levels. Farm yields are assumed to have identical expected yields and variances within a region. In this case, the average or expected yield can be calculated from regional data. The variance of farm yields can be calculated directly from the farm data or from insurance rates using an implied volatility approach. The correlation can be developed from the regional yield variance and the farm yield variance.

Specifically, let county yield be $c$ and farm yield be $y_i$ for farm $i$. The farm yields all have the same variance but are not perfectly correlated. In each year, $y_i = c + d_i$; the sum of all $d_i$ is zero in every year and the $d_i$ are uncorrelated across counterparties. Let the farm yield standard deviation be $s_y$ and the regional yield standard deviation be $s_r$. The farm-to-regional yield correlation is given by:

$$\delta_{ya} = \frac{E[y_i c]}{s_r s_y} = \frac{E[(c+d_i)c]}{s_r s_y} = \frac{E[c^2]+E[c,d_i]}{s_r s_y} = \frac{E[c^2]}{s_r s_y} = \frac{s_r^2}{s_r s_y} = \frac{s_r}{s_y},$$

where $y$ and $c$ have been reduced by their means before calculating the correlation. Consequently, the regional-to-farm yield correlation is simply the ratio of the yield standard deviations. The across-farm yield correlations can also be calculated as:

$$\begin{aligned}\delta_y &= \frac{E[y_i y_j]}{s_y s_y} = \frac{E[(c+d_i)(c+d_j)]}{s_y^2} = \frac{E[c^2+cd_i+cd_j+d_i d_j]}{s_y^2} \\ &= \frac{E[c^2]+E[cd_i]+E[cd_j]+E[d_i d_j]}{s_y^2} \\ &= \frac{E[c^2]}{s_y^2} = \frac{s_r^2}{s_y^2} = \delta_{yr}^2\end{aligned}.$$

The between-farm correlation is simply the ratio of the regional and farm variance or the square of the regional to farm correlation. These correlations only hold when there are a large number of farms (i.e., asymptotically).

**4.2.3.3. Probability Density Functions.** After calculating the portfolio statistics, the probability-density function of portfolio losses or indemnities can be developed. The usual parametric practice is to choose a flexible functional form—such as the Beta or Gamma distribution—for the density function. For example, the parameters of the Beta function can be calculated with moment matching. If the Beta function is $\beta\ [u,w]$, then

$$\begin{aligned}u &= E[I]((E[I](1-E[I])/V[I])-1) \\ w &= (1-E[I])((E[I](1-E[I])/V[I])-1)\end{aligned}.$$

Using $\beta\ [u, w]$, the VaR can be calculated to determine capital adequacy levels.

To illustrate, consider the following example. Let $E[y]$ =1, $t$ = 0.70, $V[y]$ = $0.28^2$, and yields be normally distributed. In this case, $E[I]$ = 0.0203. Assuming identical farms and correlated yields, the $E[I]$ per dollar of exposure is 2.03/70 = 0.029 and the variance of area yields is $0.20^2$. Therefore, the between-farm correlation is

$$\delta_y = \left(\frac{0.20}{0.28}\right)^2 = 0.51.$$

$E[I]$ is the portion of exposure equal to the rate—in this case,2.9%. The frequency is $F[t]$=0.142 and $f\,[t]$=0.803 which are both useful for calculating $V[I]$. Using an earlier equation based on the normal distribution, $V[I]$ = 0.00462 which is rescaled as a proportion of exposure by dividing by the square of the trigger or $0.00462/(0.70)^2$ = 0.00943. The portfolio exposure is $V[I_p]$=0.00943*0.51=0.00481.

**4.2.3.4. Confidence Intervals.** The parameter values for the $\beta[u, w]$ distribution are $u$ = 0.141 and $w$ = 4.717. If a confidence level of 0.998 is chosen, then the required capital is 52% of exposure from Table 3. The $E[I]$ is 2.9%, so VaR = 52% - 2.9% = 49.1%. There is no internal mode in this particular case, unlike in Figure 7.

Table 3. Capital Requirements

| Percent Explosure | Confidence Level | Indemnity Probability Distribution |
|---|---|---|
| 0.40 | 0.9935 | 1.3621E-09 |
| 0.41 | 0.9941 | 9.5811E-10 |
| 0.42 | 0.9946 | 6.7132E-10 |
| 0.43 | 0.9951 | 4.6844E-10 |
| 0.44 | 0.9956 | 3.2547E-10 |
| 0.45 | 0.9960 | 2.2510E-10 |
| 0.46 | 0.9964 | 1.5493E-10 |
| 0.47 | 0.9967 | 1.0610E-10 |
| 0.48 | 0.9971 | 7.2273E-11 |
| 0.49 | 0.9974 | 4.8955E-11 |
| 0.50 | 0.9976 | 3.2965E-11 |
| 0.51 | 0.9979 | 2.2061E-11 |
| 0.52 | 0.9981 | 1.4667E-11 |
| 0.53 | 0.9983 | 9.6851E-12 |
| 0.54 | 0.9985 | 6.3493E-12 |
| 0.55 | 0.9986 | 4.1310E-12 |
| 0.56 | 0.9988 | 2.6664E-12 |
| 0.57 | 0.9989 | 1.7066E-12 |
| 0.58 | 0.9990 | 1.0827E-12 |
| 0.59 | 0.9992 | 6.8053E-13 |
| 0.60 | 0.9993 | 4.2355E-13 |

**4.2.4. Empirical Monte Carlo Approach.** The Monte Carlo VaR approach is widely used by large banks and is becoming more popular with insurance companies. The procedure can be illustrated using a crop insurance example. These models often combine economy- or sector-wide data with individual data to develop empirical probability distributions for insurance company indemnities. As with the parametric approach, the Monte Carlo approach has both pros and cons. Individual counterparty data are often not available in sufficient quantity or length to capture the extreme outcomes necessary to estimate required capital for the desired confidence level. Consequently, long-term area data are also used. For crop insurance situations, long-term data are more likely available for regional yields rather than individual farm-yield data. However, shorter-term farm yields may be available and can be used to estimate relationships between farm-level yield variations and regional-yield variations. The correlation between farm yields can also be calculated using farm- and regional-level yields, as discussed earlier.

Although Monte Carlo approaches reduce demands for detailed historical data, the approach can be complicated, difficult to comprehend, and time consuming to develop, maintain, review, and understand. Such models are often viewed as "black boxes" with only the developer having a full understanding of the model. Nonetheless, this approach may hold the most potential for accurately modeling financial risk because it may better capture idiosyncrasies of counterparty risk.

The following example is provided for illustration purposes only and is not intended to be a comprehensive discussion of empirical VaR approaches. However, it presents an intuitive approach that should provide crop insurance practioners with a working knowledge of empirical VaR modeling, which is often used to develop rates in tranched reinsurance systems.

The major steps in the process include:

1. Generating simulated farm-yield vectors with the desired expected yields, yield standard deviations, and correlations.
2. Calculating indemnities for each farm's yield and aggregate those indemnities across farms to generate empirical indemnity loss probability distributions.
3. Calculating capital adequacy if all risk is retained for a desired confidence level.
4. Calculating reinsurance premiums and capital adequacy.

Table 4 presents a simplified crop insurance example with a 70% coverage level. Assume a crop insurance company operates in two regions (e.g., two counties in the United States or two oblasts in Ukraine), and each region produces two crops. Four farms exist in region 1: two farms produce crop A and two produce crop B. In region 2 there are seven farms: three farms produce crop A and four produce crop B. If a farm produces both crops, then that farm is treated as two separate entities, each producing a single crop. This treatment presumes that, for insurance purposes, the two crops are adjusted and indemnity payments are calculated separately for each crop. Assume that these farms are large and all monetary values are in millions.

**Table 4.** Expected Yields, Yield Standard Deviations, and Correlations

| Region | Crop | Crop/Area Combination Index | Number of Farms | Expected Yield | Yield Standard Deviation | |
|---|---|---|---|---|---|---|
| | | | | | Regional | Farm |
| 1 | A | 1 | 2 | 20 | 6 | 8 |
| 1 | B | 2 | 2 | 35 | 7 | 10 |
| 2 | A | 3 | 3 | 40 | 9 | 14 |
| 2 | B | 4 | 4 | 50 | 10 | 15 |

For simplicity, yields are assumed to be normally distributed, although other distributions could be used. All farms within a region/crop combination are identically distributed (i.e., they have the same mean and standard deviation) but are imperfectly correlated. The per unit price for each crop is 1. The farm yield standard deviation could be calculated from crop insurance rates using implied volatility, directly from farm yield data, or as adjustments from regional yield standard deviations (e.g., U.S. farm yield variance is often about twice county-level yield variance). Regional data can be used to calculate expected yields, standard deviation of regional yields, and the correlation between regional yields.

**4.2.4.1. Generating Simulated Farm Data.** Three steps are required to generate simulated farm yield data:

1. Generate between-farm correlations,
2. Generate random samples
3. Generate simulated farm yields.

Each of the three steps is considered in detail below.

1. *Generate between-farm correlations.* Between-farm correlations can be calculated using regional yield correlations. The farm correlation between regional/crop combinations, $r_{ij}$, is

$$r_{ij} = \sqrt{r_{ii}}\, w_{ij} \sqrt{r_{jj}}\,,$$

where $w_{ij}$ is the regional yield correlation between regional/crop combinations and $r_{ii}$ is the yield correlation between farms with same crop within a region. Recall that

$$r_{ii} = \left( \frac{s_i^r}{s_i^f} \right)^2,$$

where $s_i^r$ is the standard deviation of yields at the regional level and $s_i^f$ is the standard deviation at the farm level with *i* the area/crop index. Therefore, when *i=j*, the correlation is calculated directly from standard deviations. To calculate $r_{ij},\ i \neq j$, the previously presented equation is used. For our example, assume that

$$w_{ij} = \begin{bmatrix} 1.0 & 0.3 & 0.2 & 0.1 \\ 0.3 & 1.0 & 0.05 & 0.15 \\ 0.2 & 0.05 & 1.0 & 0.25 \\ 0.1 & 0.15 & 0.25 & 1.0 \end{bmatrix}$$

and so

$$r_{ij} = \begin{bmatrix} 0.563 & 0.158 & 0.096 & 0.050 \\ 0.158 & 0.490 & 0.023 & 0.070 \\ 0.096 & 0.023 & 0.413 & 0.107 \\ 0.050 & 0.070 & 0.107 & 0.444 \end{bmatrix}$$

2. *Generate random samples.* The next step is to generate a random sample such that the number of variables is equal to the number of farms plus the number of region/crop combinations. In this example, fifteen independent variables are generated and each is distributed as a standard normal. In most actual examples, the length of the sample is usually between 2,000 and 10,000. For purposes of exposition, we will illustrate with a data set of length 50.

Table 5 contains a random sample designated $x_{ik}$ generated using Excel. The first three variables are designated $x_{10}$ through $x_{12}$, the next three are $x_{20}$ through $x_{22}$, the next four are $x_{30}$ through $x_{33}$, and the last five are $x_{40}$ through $x_{44}$. The first subscript designates the region/crop combination., and the second subscript designates the farm if the subscript is greater than 0 and designates an "anchor" variable if the subscript is 0. Anchor variables are used for computational convenience in the early portion of this step. Most spreadsheets (such as Excel), data processing packages (such as R or SQL), and statistical packages (such as SAS) offer random-number generators.

3. *Generate simulated farm yields.* The final step is to generate a data set of $z_{ik}$, that have the desired correlations but remain distributed as a standard normal. Similar to a Cholesky decomposition, the *z* are generated from the *x* as

$$\begin{aligned} z_{1k} &= \alpha_{11}x_{10} + \alpha_{12}x_{1k} \\ z_{2k} &= \alpha_{21}x_{10} + \alpha_{22}x_{20} + \alpha_{23}x_{2k} \\ z_{3k} &= \alpha_{31}x_{10} + \alpha_{32}x_{20} + \alpha_{33}x_{30} + \alpha_{34}x_{3k} \\ z_{4k} &= \alpha_{41}x_{10} + \alpha_{42}x_{20} + \alpha_{43}x_{30} + \alpha_{44}x_{40} + \alpha_{45}x_{4k} \end{aligned}$$

Table 5. Random Sample of Xik's

| Obs | x10 | x11 | x12 | x20 | x21 | x22 | x30 | x31 | x32 | x33 | x40 | x41 | x42 | x43 | x44 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -0,32 | -1,52 | 0,16 | 0,92 | 1,06 | 1,60 | -2,38 | -0,23 | 0,99 | -1,04 | -0,74 | -1,44 | -1,87 | -1,02 | -0,88 |
| 2 | -2,16 | -0,75 | -0,48 | -0,04 | -0,43 | -0,46 | -0,54 | 1,52 | -0,04 | -0,12 | -0,57 | 1,68 | 0,82 | 2,23 | -0,75 |
| 3 | 1,65 | -1,89 | 0,44 | 0,61 | 1,74 | -0,22 | -0,70 | 0,77 | -0,29 | 0,85 | -1,48 | -0,72 | -1,55 | -0,42 | -0,02 |
| 4 | 0,01 | -0,49 | 2,06 | -1,62 | -0,78 | -2,71 | 1,31 | -1,40 | -0,53 | 0,85 | 0,40 | 0,75 | 0,55 | -1,40 | -1,28 |
| 5 | 0,68 | 0,22 | -1,00 | -0,35 | 0,04 | 0,42 | -0,02 | -0,99 | 1,67 | 0,57 | 0,01 | 0,71 | 0,81 | -0,69 | -1,06 |
| 6 | 1,10 | -1,44 | -1,60 | 0,45 | 0,52 | 2,07 | 1,30 | 1,47 | 0,14 | 0,07 | 0,38 | -0,02 | -1,09 | -1,79 | 0,98 |
| 7 | 0,43 | 0,54 | 0,13 | -1,02 | 1,09 | -0,44 | -1,02 | -0,89 | -0,33 | -0,39 | -0,58 | 0,73 | 0,47 | -0,66 | 1,54 |
| 8 | -1,80 | 0,46 | -0,20 | -0,12 | -0,70 | -0,68 | 0,70 | 0,92 | 0,43 | 0,78 | 1,54 | 0,26 | 0,54 | 1,72 | -0,37 |
| 9 | 1,02 | 0,02 | 1,03 | -0,28 | -0,82 | 0,94 | -0,76 | 0,62 | 0,51 | -0,25 | -0,49 | -1,16 | 1,93 | -0,62 | 0,12 |
| 10 | -0,26 | 2,95 | 1,14 | 0,59 | 1,18 | 0,05 | 0,39 | -0,25 | -1,02 | 1,37 | -0,35 | -1,11 | 0,71 | 0,69 | 0,52 |
| 11 | 0,39 | -0,84 | 0,62 | 0,29 | 0,94 | -2,90 | 0,31 | 1,65 | -1,46 | 0,12 | 1,27 | 1,44 | 0,19 | 0,07 | 2,18 |
| 12 | -0,06 | -0,85 | 0,80 | -0,16 | 0,90 | 0,30 | 1,31 | -0,11 | -0,80 | -1,50 | -2,58 | 0,09 | 0,25 | -0,09 | 0,25 |
| 13 | 0,24 | 1,47 | 0,00 | -1,09 | -1,40 | 0,06 | 0,80 | -1,65 | -0,49 | 0,61 | -0,27 | -0,29 | -0,20 | -0,50 | -0,92 |
| 14 | 0,63 | -0,07 | 1,15 | 0,07 | -0,03 | 0,09 | 0,05 | 0,02 | -1,02 | 0,31 | 0,08 | 0,64 | -0,77 | 0,53 | 0,27 |
| 15 | 0,09 | 0,81 | 1,67 | -1,25 | -1,26 | 0,42 | 1,05 | -0,95 | -1,82 | -0,85 | 0,14 | 0,37 | -1,02 | -0,90 | 0,56 |
| 16 | 0,10 | -0,72 | 0,02 | -0,94 | -0,93 | 0,91 | 1,49 | -0,72 | -0,11 | 0,19 | 1,75 | 0,51 | 1,60 | 0,10 | 1,84 |
| 17 | -0,67 | -1,18 | 1,08 | 0,62 | -1,73 | 1,14 | -0,37 | -1,23 | -1,59 | -0,77 | 0,40 | 0,78 | -0,11 | -0,12 | 0,27 |
| 18 | -0,38 | 0,87 | -0,56 | 1,82 | 0,62 | 0,48 | -1,16 | 0,99 | 1,29 | -0,68 | -0,36 | -1,14 | 1,30 | 0,94 | 0,23 |
| 19 | -2,23 | 0,41 | 0,43 | 0,87 | -0,99 | 1,07 | -0,74 | -0,79 | -1,46 | 1,48 | -1,01 | 1,32 | -1,20 | -0,47 | -1,51 |
| 20 | 1,58 | -0,10 | 0,24 | 0,67 | 1,77 | -0,19 | -1,21 | -0,61 | 0,77 | 0,96 | 0,44 | -1,26 | -1,43 | -0,21 | -0,72 |
| 21 | -0,65 | 0,82 | -0,90 | -0,25 | -0,30 | 1,38 | -1,50 | -1,01 | -0,11 | -1,02 | -1,47 | -1,84 | 0,53 | -0,40 | 0,50 |
| 22 | -2,09 | 1,13 | -1,98 | -0,59 | -1,01 | -0,23 | -0,03 | 1,95 | -0,40 | -0,74 | -0,80 | 0,50 | 1,23 | -1,46 | 1,15 |
| 23 | 1,58 | -0,86 | 0,00 | -0,85 | 0,58 | 0,10 | -0,12 | -1,12 | -0,22 | 0,81 | -0,07 | 0,86 | -1,05 | 0,66 | 1,47 |
| 24 | -1,20 | 0,30 | -1,82 | -0,08 | 0,87 | 0,90 | -0,06 | -0,82 | 0,54 | 0,55 | 0,65 | -0,81 | 0,55 | 1,49 | 0,29 |
| 25 | -0,41 | 0,14 | 1,69 | -2,17 | -0,07 | -0,26 | 0,84 | 1,40 | 1,46 | -1,79 | 0,73 | 0,31 | 0,69 | 0,40 | -0,14 |
| 26 | 1,65 | 0,36 | -0,11 | -0,17 | -0,13 | -1,81 | 0,29 | 0,24 | 0,60 | -0,10 | 0,75 | 1,13 | 1,84 | -1,34 | 0,74 |
| 27 | -0,24 | -0,94 | 0,64 | 1,81 | 1,20 | 0,05 | -0,65 | -0,24 | -0,91 | 0,15 | -0,32 | 0,55 | -1,30 | 0,43 | 1,16 |
| 28 | -0,89 | 0,27 | 0,44 | 0,67 | -0,30 | 0,10 | -0,59 | 0,59 | 1,41 | -0,09 | -1,07 | -0,78 | 0,68 | 1,48 | -0,72 |
| 29 | -0,86 | 0,98 | 0,14 | 0,23 | 2,39 | -0,71 | -0,96 | 1,37 | -0,23 | -1,38 | 1,04 | -1,10 | -0,44 | -1,47 | -1,26 |
| 30 | 0,59 | 0,00 | -1,79 | 1,56 | 0,15 | 1,29 | -0,19 | -0,24 | 0,06 | -1,83 | -0,74 | 1,18 | 0,12 | -0,50 | -0,69 |
| 31 | -1,18 | 1,22 | 2,33 | 0,62 | -0,18 | -0,99 | -0,25 | -0,26 | -0,30 | 0,96 | -0,34 | 0,30 | 0,75 | 0,16 | -1,88 |
| 32 | 0,01 | 0,57 | -0,24 | -1,54 | -1,60 | -0,94 | 0,10 | -0,65 | 1,83 | -0,15 | 0,79 | 1,06 | -0,50 | -1,86 | -1,03 |
| 33 | 0,08 | 0,16 | 0,79 | -1,90 | -0,73 | 0,02 | -0,02 | -0,06 | -0,17 | 1,57 | 0,76 | -1,13 | -0,51 | -0,32 | -1,07 |
| 34 | -0,22 | 0,03 | -1,02 | 0,56 | -0,25 | 0,57 | -0,93 | 0,37 | -1,05 | 2,41 | 0,75 | -0,96 | 1,44 | -0,34 | -1,64 |
| 35 | -0,46 | 0,17 | -0,77 | 0,34 | -0,31 | 0,39 | -2,12 | -1,21 | 1,33 | 0,28 | 0,82 | -1,04 | -1,30 | -0,31 | -0,20 |
| 36 | 1,00 | 0,97 | -0,36 | 0,27 | 0,82 | -0,90 | -0,92 | -0,31 | 0,48 | -1,69 | 1,12 | 1,64 | -0,03 | -1,32 | 0,04 |
| 37 | 1,47 | 0,04 | 1,15 | 0,74 | 0,27 | -0,89 | -0,61 | 1,51 | 0,43 | 0,79 | -0,99 | -1,79 | -0,09 | 0,90 | 1,07 |
| 38 | -0,52 | -0,87 | 0,20 | -1,05 | 0,43 | -1,73 | 1,31 | 1,82 | -0,54 | 1,38 | 0,63 | 0,79 | -1,13 | 1,16 | -0,49 |
| 39 | 1,24 | -1,87 | -0,96 | 2,39 | -0,49 | -0,55 | 0,52 | -0,32 | -1,47 | 0,12 | 0,82 | 0,77 | -0,30 | 0,36 | 2,13 |
| 40 | 0,43 | -0,04 | -0,61 | 0,09 | -0,10 | 1,45 | 2,79 | -2,01 | 0,16 | 1,79 | 0,88 | 0,17 | 1,10 | 0,91 | -0,11 |
| 41 | -0,81 | -1,22 | -1,21 | -2,27 | -1,22 | 0,53 | 0,65 | 1,36 | -0,34 | -1,22 | -1,90 | -0,41 | -0,41 | 0,87 | -0,73 |
| 42 | 1,01 | 0,19 | 0,79 | 0,81 | 0,51 | 0,15 | -0,34 | 0,47 | -0,43 | 0,91 | -0,16 | -0,59 | -0,88 | 1,65 | 0,49 |
| 43 | 0,05 | 1,00 | -1,00 | -0,11 | -0,26 | 0,38 | 1,67 | -0,44 | 1,33 | 0,69 | 1,51 | 1,73 | 0,86 | 1,86 | -0,04 |
| 44 | 0,00 | 1,52 | -0,74 | -0,01 | -1,14 | -0,15 | 1,27 | 0,39 | 0,11 | -0,72 | -1,95 | -0,48 | -1,67 | 0,96 | 0,01 |
| 45 | 1,16 | 1,09 | 0,79 | 0,36 | 0,66 | 0,16 | -0,24 | -0,35 | -0,53 | -0,05 | 1,77 | 0,00 | 1,46 | -0,77 | 2,04 |
| 46 | -1,28 | 0,30 | 0,37 | -1,06 | 2,32 | 0,46 | -1,21 | -1,59 | -1,39 | -0,38 | -1,63 | -1,08 | -1,81 | -0,75 | -0,33 |
| 47 | 1,05 | 0,67 | -0,23 | -0,28 | -0,94 | 1,18 | 0,87 | -0,60 | 1,50 | -0,92 | 0,73 | 1,24 | 0,30 | -0,74 | -0,22 |
| 48 | 0,69 | -2,15 | -0,91 | 0,14 | -0,44 | -1,33 | 0,09 | 0,86 | -1,44 | -0,38 | -0,81 | 0,34 | -0,25 | -0,56 | 0,22 |
| 49 | -0,99 | -1,15 | -0,47 | 1,47 | -1,83 | -0,27 | 0,21 | 0,33 | 1,39 | -1,65 | 0,22 | -0,87 | 0,35 | 0,61 | -0,98 |
| 50 | -0,24 | -0,71 | -1,33 | 0,18 | 0,32 | -0,27 | 0,33 | 0,41 | 2,06 | -0,81 | 0,31 | -1,80 | -0,17 | 0,88 | -1,00 |
| **Mean** | **0,00** | **0,00** | **0,00** | **0,00** | **0,00** | **0,00** | **0,00** | **0,00** | **0,00** | **0,00** | **0,00** | **0,00** | **0,00** | **0,00** | **0,00** |
| **Std Dev** | **1,00** | **1,00** | **1,00** | **1,00** | **1,00** | **1,00** | **1,00** | **1,00** | **1,00** | **1,00** | **1,00** | **1,00** | **1,00** | **1,00** | **1,00** |

The values for $\alpha$ are developed below. The variance of $z$ and $x$ is 1, which simplifies the calculations. The squares of $\alpha$ must sum to 1 for each equation to maintain the variance of 1 for the $z$.

$$r_{11} = E[z_{1k}z_{1v}] = E[(\alpha_{11}x_{10} + \alpha_{12}x_{1k})(\alpha_{11}x_{10} + \alpha_{12}x_{1v})] = \alpha_{11}^2 \Rightarrow \alpha_{11} = \sqrt{r_{11}}$$
$$1 = \alpha_{11}^2 + \alpha_{12}^2 \Rightarrow \alpha_{12} = \sqrt{1-\alpha_{11}^2}$$
$$r_{21} = E[z_{1k}z_{2v}] = E[(\alpha_{11}x_{10} + \alpha_{12}x_{1k})(\alpha_{21}x_{20} + \alpha_{22}x_{20} + \alpha_{23}x_{2v})] = \alpha_{11}\alpha_{21} \Rightarrow \alpha_{21} = {r_{21}}/{\alpha_{11}}$$
$$r_{22} = E[z_{2k}z_{2v}] = E[(\alpha_{21}x_{20} + \alpha_{22}x_{20} + \alpha_{23}x_{2k})(\alpha_{21}x_{20} + \alpha_{22}x_{20} + \alpha_{23}x_{2v})] = \alpha_{21}^2 + \alpha_{22}^2 \Rightarrow \alpha_{22} = \sqrt{r_{22} - \alpha_{21}^2}$$
$$1 = \alpha_{21}^2 + \alpha_{22}^2 + \alpha_{23}^2 \Rightarrow \alpha_{23} = \sqrt{1-\alpha_{21}^2-\alpha_{22}^2}$$

The process is continued for all of the region/crop combinations. A matrix of $\alpha$ presents this example's four region/crop combinations.

$$\begin{bmatrix} \sqrt{r_{11}} & \sqrt{1-\alpha_{11}^2} & 0 & 0 & 0 \\ {r_{21}}/{\alpha_{11}} & \sqrt{r_{22}-\alpha_{21}^2} & \sqrt{1-\alpha_{21}^2-\alpha_{22}^2} & 0 & 0 \\ {r_{31}}/{\alpha_{11}} & {(r_{32}-\alpha_{21}\alpha_{31})}/{\alpha_{22}} & \sqrt{r_{33}-\alpha_{31}^2-\alpha_{32}^2} & \sqrt{1-\alpha_{31}^2-\alpha_{32}^2-\alpha_{33}^2} & 0 \\ {r_{41}}/{\alpha_{11}} & {(r_{42}-\alpha_{21}\alpha_{41})}/{\alpha_{22}} & {(r_{43}-\alpha_{31}\alpha_{41}-\alpha_{32}\alpha_{42})}/{\alpha_{33}} & \sqrt{r_{44}-\alpha_{41}^2-\alpha_{42}^2-\alpha_{43}^2} & \sqrt{1-\alpha_{41}^2-\alpha_{42}^2-\alpha_{43}^2-\alpha_{44}^2} \end{bmatrix}$$

In the example, the $\alpha_{ik}$ are calculated as

$$\alpha_{ik} = \begin{bmatrix} 0.750 & 0.661 & 0 & 0 & 0 \\ 0.210 & 0.668 & 0.714 & 0 & 0 \\ 0.129 & -0.007 & 0.630 & 0.766 & 0 \\ 0.067 & 0.030 & 0.157 & 0.644 & 0.745 \end{bmatrix}$$

The $z_{ik}$ are calculated using the $x_{ik}$ and the $\alpha_{ik}$'s. The number of $z$ variables is equal to the total number of farms (eleven in this example). The $z_{ik}$ are presented in Table 6.

The $z$ variables are transformed to simulated yields using

$$y_{ik} = E[y_i] + z_{ik}s_i,$$

where $E[y_i]$ is the expected yield for region/crop $i$ and $s_i$ is the standard deviation of yields for region/crop $i$. The $y$ values for the example are presented in Table 7. Yields $y$ have the same expected yield and variance as in Table 4 as well as the desired correlations.

**Table 6.** Transformed Z's

| Obs | z11 | z12 | z21 | z22 | z31 | z32 | z33 | z41 | z42 | z43 | z44 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -1,25 | -0,14 | 1,30 | 1,69 | -1,73 | -0,79 | -2,35 | -1,92 | -2,24 | -1,60 | -1,50 |
| 2 | -2,11 | -1,93 | -0,78 | -0,81 | 0,54 | -0,64 | -0,71 | 0,66 | 0,01 | 1,07 | -1,15 |
| 3 | -0,01 | 1,53 | 2,00 | 0,60 | 0,36 | -0,45 | 0,42 | -1,47 | -2,09 | -1,25 | -0,95 |
| 4 | -0,32 | 1,37 | -1,64 | -3,01 | -0,23 | 0,43 | 1,48 | 0,97 | 0,82 | -0,63 | -0,54 |
| 5 | 0,65 | -0,15 | -0,06 | 0,21 | -0,68 | 1,36 | 0,51 | 0,57 | 0,64 | -0,48 | -0,75 |
| 6 | -0,13 | -0,24 | 0,90 | 2,01 | 2,09 | 1,06 | 1,01 | 0,52 | -0,27 | -0,80 | 1,27 |
| 7 | 0,67 | 0,40 | 0,19 | -0,91 | -1,26 | -0,84 | -0,88 | 0,01 | -0,19 | -1,03 | 0,61 |
| 8 | -1,04 | -1,48 | -0,96 | -0,94 | 0,91 | 0,54 | 0,81 | 1,17 | 1,38 | 2,26 | 0,70 |
| 9 | 0,78 | 1,45 | -0,56 | 0,70 | 0,13 | 0,05 | -0,54 | -1,24 | 1,06 | -0,84 | -0,29 |
| 10 | 1,76 | 0,56 | 1,19 | 0,38 | 0,01 | -0,58 | 1,25 | -0,99 | 0,37 | 0,35 | 0,22 |
| 11 | -0,27 | 0,70 | 0,95 | -1,79 | 1,51 | -0,87 | 0,33 | 1,97 | 1,04 | 0,95 | 2,52 |
| 12 | -0,61 | 0,48 | 0,53 | 0,09 | 0,74 | 0,21 | -0,33 | -1,40 | -1,28 | -1,53 | -1,28 |
| 13 | 1,15 | 0,18 | -1,68 | -0,64 | -0,72 | 0,17 | 1,01 | -0,28 | -0,22 | -0,44 | -0,75 |
| 14 | 0,42 | 1,23 | 0,16 | 0,24 | 0,13 | -0,67 | 0,35 | 0,59 | -0,47 | 0,50 | 0,31 |
| 15 | 0,60 | 1,18 | -1,71 | -0,51 | -0,05 | -0,72 | 0,03 | 0,49 | -0,54 | -0,45 | 0,63 |
| 16 | -0,40 | 0,09 | -1,28 | 0,04 | 0,41 | 0,87 | 1,10 | 1,72 | 2,53 | 1,41 | 2,71 |
| 17 | -1,29 | 0,21 | -0,96 | 1,09 | -1,27 | -1,54 | -0,91 | 0,76 | 0,09 | 0,08 | 0,38 |
| 18 | 0,30 | -0,66 | 1,58 | 1,48 | -0,03 | 0,20 | -1,31 | -1,23 | 0,58 | 0,32 | -0,21 |
| 19 | -1,40 | -1,39 | -0,59 | 0,88 | -1,36 | -1,87 | 0,38 | 0,10 | -1,78 | -1,24 | -2,01 |
| 20 | 1,12 | 1,34 | 2,04 | 0,64 | -1,03 | 0,03 | 0,17 | -0,72 | -0,84 | 0,06 | -0,32 |
| 21 | 0,05 | -1,09 | -0,52 | 0,68 | -1,81 | -1,11 | -1,81 | -2,61 | -0,84 | -1,53 | -0,86 |
| 22 | -0,81 | -2,88 | -1,55 | -1,00 | 1,21 | -0,59 | -0,85 | -0,30 | 0,24 | -1,76 | 0,18 |
| 23 | 0,62 | 1,18 | 0,18 | -0,16 | -0,72 | -0,03 | 0,76 | 0,65 | -0,76 | 0,51 | 1,11 |
| 24 | -0,70 | -2,10 | 0,32 | 0,33 | -0,82 | 0,22 | 0,23 | -0,27 | 0,74 | 1,44 | 0,55 |
| 25 | -0,22 | 0,81 | -1,58 | -1,72 | 1,56 | 1,61 | -0,88 | 0,74 | 1,02 | 0,80 | 0,40 |
| 26 | 1,48 | 1,16 | 0,14 | -1,06 | 0,58 | 0,86 | 0,32 | 1,47 | 2,00 | -0,37 | 1,18 |
| 27 | -0,80 | 0,24 | 2,02 | 1,19 | -0,63 | -1,15 | -0,34 | 0,14 | -1,24 | 0,06 | 0,60 |
| 28 | -0,49 | -0,38 | 0,05 | 0,33 | -0,04 | 0,59 | -0,56 | -1,40 | -0,31 | 0,28 | -1,35 |
| 29 | 0,00 | -0,56 | 1,68 | -0,54 | 0,33 | -0,89 | -1,77 | -0,35 | 0,14 | -0,63 | -0,48 |
| 30 | 0,44 | -0,74 | 1,27 | 2,09 | -0,24 | -0,01 | -1,45 | 0,46 | -0,33 | -0,79 | -0,93 |
| 31 | -0,07 | 0,66 | 0,04 | -0,54 | -0,51 | -0,55 | 0,42 | -0,10 | 0,24 | -0,20 | -1,72 |
| 32 | 0,38 | -0,15 | -2,17 | -1,70 | -0,43 | 1,47 | -0,04 | 1,26 | 0,11 | -0,91 | -0,29 |
| 33 | 0,16 | 0,58 | -1,77 | -1,24 | -0,03 | -0,12 | 1,22 | -0,41 | 0,05 | 0,19 | -0,36 |
| 34 | -0,15 | -0,84 | 0,15 | 0,73 | -0,34 | -1,43 | 1,22 | -0,37 | 1,41 | 0,09 | -0,88 |
| 35 | -0,23 | -0,85 | -0,09 | 0,41 | -2,32 | -0,38 | -1,18 | -0,60 | -0,80 | -0,06 | 0,02 |
| 36 | 1,39 | 0,51 | 0,98 | -0,25 | -0,69 | -0,09 | -1,75 | 1,87 | 0,63 | -0,34 | 0,68 |
| 37 | 1,13 | 1,86 | 0,99 | 0,16 | 0,96 | 0,13 | 0,40 | -1,95 | -0,68 | 0,06 | 0,19 |
| 38 | -0,96 | -0,26 | -0,50 | -2,04 | 2,17 | 0,36 | 1,83 | 1,13 | -0,30 | 1,40 | 0,18 |
| 39 | -0,31 | 0,30 | 1,51 | 1,46 | 0,22 | -0,66 | 0,56 | 1,33 | 0,54 | 1,03 | 2,35 |
| 40 | 0,29 | -0,08 | 0,08 | 1,19 | 0,27 | 1,93 | 3,18 | 1,16 | 1,86 | 1,72 | 0,96 |
| 41 | -1,41 | -1,40 | -2,56 | -1,30 | 1,37 | 0,06 | -0,61 | -1,55 | -1,55 | -0,60 | -1,79 |
| 42 | 0,89 | 1,28 | 1,12 | 0,86 | 0,27 | -0,42 | 0,61 | -0,50 | -0,72 | 1,16 | 0,30 |
| 43 | 0,69 | -0,62 | -0,25 | 0,21 | 0,73 | 2,08 | 1,59 | 2,52 | 1,88 | 2,62 | 1,20 |
| 44 | 1,01 | -0,49 | -0,82 | -0,12 | 1,09 | 0,88 | 0,24 | -1,41 | -2,30 | -0,34 | -1,05 |
| 45 | 1,59 | 1,39 | 0,96 | 0,60 | -0,28 | -0,42 | -0,05 | 1,19 | 2,28 | 0,61 | 2,71 |
| 46 | -0,76 | -0,72 | 0,68 | -0,65 | -2,14 | -1,99 | -1,22 | -2,17 | -2,71 | -1,92 | -1,61 |
| 47 | 1,23 | 0,64 | -0,64 | 0,88 | 0,22 | 1,83 | -0,02 | 1,59 | 0,89 | 0,11 | 0,50 |
| 48 | -0,90 | -0,08 | -0,08 | -0,71 | 0,80 | -0,96 | -0,14 | -0,20 | -0,64 | -0,88 | -0,29 |
| 49 | -1,51 | -1,06 | -0,53 | 0,57 | 0,25 | 1,06 | -1,27 | -0,50 | 0,41 | 0,60 | -0,58 |
| 50 | -0,65 | -1,06 | 0,30 | -0,12 | 0,49 | 1,75 | -0,45 | -1,11 | 0,11 | 0,90 | -0,51 |
| **Mean** | **0,00** | **0,00** | **0,00** | **0,00** | **0,00** | **0,00** | **0,00** | **0,00** | **0,00** | **0,00** | **0,00** |
| **Std Dev** | **0,91** | **1,04** | **1,15** | **1,08** | **0,99** | **0,98** | **1,07** | **1,19** | **1,18** | **1,02** | **1,13** |

**Table 7.** Transformed Yields.

| Obs | y11 | y12 | y21 | y22 | y31 | y32 | y33 | y41 | y42 | y43 | y44 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 10,00 | 18,88 | 48,03 | 51,88 | 15,79 | 28,91 | 7,13 | 21,25 | 16,41 | 25,94 | 27,45 |
| 2 | 3,09 | 4,55 | 27,15 | 26,94 | 47,60 | 30,98 | 30,06 | 59,90 | 50,20 | 66,04 | 32,75 |
| 3 | 19,93 | 32,26 | 54,97 | 40,98 | 45,08 | 33,64 | 45,84 | 27,94 | 18,67 | 31,26 | 35,77 |
| 4 | 17,47 | 30,92 | 18,62 | 4,89 | 36,73 | 46,03 | 60,76 | 64,53 | 62,31 | 40,51 | 41,83 |
| 5 | 25,21 | 18,79 | 34,36 | 37,08 | 30,48 | 59,00 | 47,15 | 58,48 | 59,62 | 42,85 | 38,71 |
| 6 | 18,97 | 18,11 | 44,04 | 55,06 | 69,21 | 54,89 | 54,20 | 57,84 | 45,91 | 38,03 | 69,06 |
| 7 | 25,40 | 23,22 | 36,93 | 25,94 | 22,38 | 28,31 | 27,66 | 50,12 | 47,22 | 34,62 | 59,22 |
| 8 | 11,68 | 8,19 | 25,42 | 25,59 | 52,77 | 47,60 | 51,28 | 67,57 | 70,71 | 83,94 | 60,48 |
| 9 | 26,26 | 31,60 | 29,43 | 42,02 | 41,83 | 40,70 | 32,49 | 31,35 | 65,94 | 37,40 | 45,66 |
| 10 | 34,04 | 24,48 | 46,86 | 38,79 | 40,19 | 31,94 | 57,54 | 35,15 | 55,57 | 55,28 | 53,36 |
| 11 | 17,87 | 25,61 | 44,46 | 17,07 | 61,10 | 27,76 | 44,63 | 79,58 | 65,65 | 64,25 | 87,85 |
| 12 | 15,12 | 23,86 | 40,26 | 35,94 | 50,31 | 42,95 | 35,35 | 29,04 | 30,87 | 27,04 | 30,85 |
| 13 | 29,21 | 21,44 | 18,19 | 28,63 | 29,92 | 42,35 | 54,11 | 45,73 | 46,70 | 43,42 | 38,72 |
| 14 | 23,39 | 29,85 | 36,58 | 37,45 | 41,78 | 30,63 | 44,93 | 58,78 | 42,96 | 57,47 | 54,65 |
| 15 | 24,84 | 29,42 | 17,86 | 29,87 | 39,33 | 29,97 | 40,41 | 57,42 | 41,92 | 43,19 | 59,52 |
| 16 | 16,79 | 20,70 | 22,24 | 35,41 | 45,67 | 52,22 | 55,39 | 75,76 | 87,93 | 71,19 | 90,61 |
| 17 | 9,71 | 21,70 | 25,42 | 45,86 | 22,26 | 18,43 | 27,19 | 61,40 | 51,39 | 51,25 | 55,63 |
| 18 | 22,37 | 14,76 | 50,84 | 49,77 | 39,55 | 42,79 | 21,63 | 31,51 | 58,77 | 54,78 | 46,86 |
| 19 | 8,82 | 8,90 | 29,05 | 43,78 | 20,96 | 13,79 | 45,26 | 51,43 | 23,28 | 31,45 | 19,83 |
| 20 | 28,94 | 30,75 | 55,44 | 41,41 | 25,60 | 40,40 | 42,43 | 39,13 | 37,34 | 50,92 | 45,27 |
| 21 | 20,42 | 11,32 | 29,81 | 41,84 | 14,72 | 24,45 | 14,69 | 10,90 | 37,46 | 27,03 | 37,08 |
| 22 | 13,48 | -3,00 | 19,48 | 25,03 | 56,98 | 31,76 | 28,09 | 45,52 | 53,65 | 23,55 | 52,74 |
| 23 | 24,92 | 29,46 | 36,78 | 33,40 | 29,90 | 39,52 | 50,60 | 59,80 | 38,53 | 57,65 | 66,62 |
| 24 | 14,37 | 3,18 | 38,16 | 38,34 | 28,55 | 43,14 | 43,19 | 45,91 | 61,07 | 71,60 | 58,19 |
| 25 | 18,26 | 26,48 | 19,16 | 17,80 | 61,87 | 62,52 | 27,68 | 61,13 | 65,36 | 62,06 | 56,05 |
| 26 | 31,80 | 29,31 | 36,40 | 24,40 | 48,07 | 51,98 | 44,41 | 72,09 | 80,07 | 44,47 | 67,76 |
| 27 | 13,59 | 21,94 | 55,17 | 46,93 | 31,14 | 23,91 | 35,28 | 52,12 | 31,42 | 50,83 | 58,93 |
| 28 | 16,07 | 16,95 | 35,47 | 38,31 | 39,47 | 48,21 | 32,13 | 28,96 | 45,33 | 54,22 | 29,68 |
| 29 | 20,00 | 15,55 | 51,77 | 29,63 | 44,65 | 27,53 | 15,16 | 44,73 | 52,11 | 40,60 | 42,87 |
| 30 | 23,52 | 14,08 | 47,72 | 55,90 | 36,65 | 39,91 | 19,65 | 56,90 | 45,09 | 38,14 | 36,01 |
| 31 | 19,41 | 25,25 | 35,41 | 29,60 | 32,85 | 32,36 | 45,92 | 48,52 | 53,58 | 47,03 | 24,18 |
| 32 | 23,06 | 18,76 | 13,29 | 17,99 | 34,02 | 60,64 | 39,41 | 68,97 | 51,58 | 36,34 | 45,58 |
| 33 | 21,31 | 24,65 | 17,27 | 22,63 | 39,53 | 38,32 | 57,02 | 43,85 | 50,80 | 52,90 | 44,53 |
| 34 | 18,82 | 13,29 | 36,49 | 42,34 | 35,30 | 20,03 | 57,14 | 44,39 | 71,14 | 51,33 | 36,76 |
| 35 | 18,13 | 13,17 | 34,14 | 39,14 | 7,50 | 34,69 | 23,48 | 40,95 | 38,07 | 49,10 | 50,34 |
| 36 | 31,14 | 24,08 | 44,78 | 32,49 | 30,40 | 38,81 | 15,55 | 78,09 | 59,41 | 44,97 | 60,18 |
| 37 | 29,04 | 34,89 | 44,93 | 36,64 | 53,38 | 41,82 | 45,64 | 20,80 | 39,77 | 50,84 | 52,79 |
| 38 | 12,30 | 17,92 | 29,95 | 14,56 | 70,31 | 45,01 | 65,55 | 66,97 | 45,53 | 71,06 | 52,68 |
| 39 | 17,54 | 22,36 | 50,06 | 49,63 | 43,13 | 30,79 | 47,86 | 70,02 | 58,16 | 65,52 | 85,26 |
| 40 | 22,34 | 19,33 | 35,76 | 46,87 | 43,82 | 67,06 | 84,56 | 67,46 | 77,87 | 75,80 | 64,38 |
| 41 | 8,70 | 8,78 | 9,43 | 21,96 | 59,14 | 40,83 | 31,44 | 26,72 | 26,76 | 41,07 | 23,17 |
| 42 | 27,10 | 30,27 | 46,18 | 43,59 | 43,80 | 34,11 | 48,51 | 42,46 | 39,23 | 67,46 | 54,51 |
| 43 | 25,56 | 15,00 | 32,53 | 37,08 | 50,16 | 69,15 | 62,24 | 87,85 | 78,17 | 89,31 | 68,06 |
| 44 | 28,04 | 16,07 | 26,84 | 33,84 | 55,31 | 52,29 | 43,41 | 28,82 | 15,43 | 44,87 | 34,23 |
| 45 | 32,74 | 31,14 | 44,57 | 40,97 | 36,12 | 34,19 | 39,35 | 67,86 | 84,17 | 59,19 | 90,65 |
| 46 | 13,89 | 14,26 | 41,80 | 28,50 | 9,99 | 12,14 | 22,96 | 17,51 | 9,31 | 21,18 | 25,88 |
| 47 | 29,83 | 25,10 | 28,62 | 43,77 | 43,13 | 65,64 | 39,70 | 73,86 | 63,33 | 51,72 | 57,50 |
| 48 | 12,78 | 19,34 | 34,22 | 27,90 | 51,26 | 26,53 | 38,00 | 46,99 | 40,34 | 36,83 | 45,61 |
| 49 | 7,94 | 11,53 | 29,65 | 40,74 | 43,44 | 54,88 | 22,20 | 42,52 | 56,19 | 59,04 | 41,35 |
| 50 | 14,81 | 11,54 | 38,01 | 33,82 | 46,86 | 64,49 | 33,72 | 33,41 | 51,69 | 63,45 | 42,38 |
| **Mean** | **20,00** | **20,00** | **35,00** | **35,00** | **40,00** | **40,00** | **40,00** | **50,00** | **50,00** | **50,00** | **50,00** |

**4.2.4.2. Calculating Simulated Indemnities.** The next major step is to calculate simulated indemnities for each of the eleven simulated farm yields. The indemnities are calculated as $I = t - y$, where $t$ is the trigger (calculated as expected yield multiplied by the 70% coverage level). If $t - y$ is negative, then $I = 0$ because no indemnities are paid when $y > t$. After indemnities are calculated, they are summed across "years" to calculate annual total indemnities. Indemnities for the example are presented in Table 8. The annual indemnity will often be divided by the total exposure or total liability to calculate the annual liability paid as a percent of exposure (subsequently referred to as the percent liability or percent of exposure). In the example, total exposure is

$$(2)\ (20)\ (0.7) + (2)\ (35)\ (0.7) + (3)(40)(0.7) + (4)(50)(0.7) = 301 \text{ million}$$

Therefore, 301 million would be divided into each year's indemnity payment. Table 8 includes each year's percent liability and also the percent liability ranked according to size. The ranked percent liability makes it easier to visualize the probability distribution of indemnities. A histogram is often useful. The percent liability in the example is somewhat misleading because if the number of counterparties was increased to a more realistic number, the number of cells containing zeros would decline. This is simply a pdf of indemnities as a percentage of liability, which directly relates to Figure 7.

**4.2.4.3. Calculating Capital Adequacy for Risk Retention.** For demonstration purposes, the VaR plus $E[I]$ (or the required capital) at the 90% confidence level equals 18.8% - 19% (Table 8). $E[I]$ can be calculated either by averaging the indemnities or from the pure risk insurance rates. In this example, $E[I]$ = 5.3%, which results in a VaR of 13.5% (18.8% - 5.3%). Consequently, required capital needed if all risk responsibility (i.e., indemnity payments) is retained is 18.8% of the total liability or 56.9 million (0.188 * 301 million).

Some cautionary comments are appropriate. The example presented above is used to illustrate the Monte Carlo process. In the example, no indemnity payments are made in several years. This would be unlikely in situations where there are a large number of counterparties. The lack of indemnities in some years biases the VaR calculation downward relative to an actual insurance situation. In addition, the number of years in this illustration was limited to fifty for ease of presentation. In practice, however, many more years of data must be simulated. The confidence level was set at 0.90 because of the limited number of years presented. In practice, confidence levels are often set at 0.992 or higher. Finally, the required capital calculations do not include operations capital and other considerations.

**Table 8.** Simulated Indemnities and Proportion of Total Liability.

| Trigger | 14 | 14 | 24.5 | 24.5 | 28 | 28 | 28 | 35 | 35 | 35 | 35 | Sum of Indemni-ties | Proportion Indemnity of Total Liability | Ranked Proportion Indemnity of Total Liability |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Obs | I11 | I12 | I21 | I22 | I31 | I32 | I33 | I41 | I42 | I43 | I44 | | | |
| 1 | 4,00 | 0,00 | 0,00 | 0,00 | 12,21 | 0,00 | 20,87 | 13,75 | 18,59 | 9,06 | 7,55 | 86,03 | 0,29 | 0,35 |
| 2 | 10,91 | 9,45 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 2,25 | 22,62 | 0,08 | 0,29 |
| 3 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 7,06 | 16,33 | 3,74 | 0,00 | 27,13 | 0,09 | 0,22 |
| 4 | 0,00 | 0,00 | 5,88 | 19,61 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 25,49 | 0,08 | 0,21 |
| 5 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,19 |
| 6 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,11 |
| 7 | 0,00 | 0,00 | 0,00 | 0,00 | 5,62 | 0,00 | 0,34 | 0,00 | 0,00 | 0,38 | 0,00 | 6,34 | 0,02 | 0,09 |
| 8 | 2,32 | 5,81 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 8,14 | 0,03 | 0,09 |
| 9 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 3,65 | 0,00 | 0,00 | 0,00 | 3,65 | 0,01 | 0,09 |
| 10 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,08 |
| 11 | 0,00 | 0,00 | 0,00 | 7,43 | 0,00 | 0,24 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 7,67 | 0,03 | 0,08 |
| 12 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 5,96 | 4,13 | 7,96 | 4,15 | 22,20 | 0,07 | 0,07 |
| 13 | 0,00 | 0,00 | 6,31 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 6,31 | 0,02 | 0,07 |
| 14 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,06 |
| 15 | 0,00 | 0,00 | 6,64 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 6,64 | 0,02 | 0,05 |
| 16 | 0,00 | 0,00 | 2,26 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 2,26 | 0,01 | 0,05 |
| 17 | 4,29 | 0,00 | 0,00 | 0,00 | 5,74 | 9,57 | 0,81 | 0,00 | 0,00 | 0,00 | 0,00 | 20,41 | 0,07 | 0,04 |
| 18 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 6,37 | 3,49 | 0,00 | 0,00 | 0,00 | 9,86 | 0,03 | 0,04 |
| 19 | 5,18 | 5,10 | 0,00 | 0,00 | 7,04 | 14,21 | 0,00 | 0,00 | 11,72 | 3,55 | 15,17 | 61,99 | 0,21 | 0,04 |
| 20 | 0,00 | 0,00 | 0,00 | 0,00 | 2,40 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 2,40 | 0,01 | 0,04 |
| 21 | 0,00 | 2,68 | 0,00 | 0,00 | 13,28 | 3,55 | 13,31 | 24,10 | 0,00 | 7,97 | 0,00 | 64,88 | 0,22 | 0,04 |
| 22 | 0,52 | 17,00 | 5,02 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 11,45 | 0,00 | 33,99 | 0,11 | 0,04 |
| 23 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,04 |
| 24 | 0,00 | 10,82 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 10,82 | 0,04 | 0,03 |
| 25 | 0,00 | 0,00 | 5,34 | 6,70 | 0,00 | 0,00 | 0,32 | 0,00 | 0,00 | 0,00 | 0,00 | 12,36 | 0,04 | 0,03 |
| 26 | 0,00 | 0,00 | 0,00 | 0,10 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,10 | 0,00 | 0,03 |
| 27 | 0,41 | 0,00 | 0,00 | 0,00 | 0,00 | 4,09 | 0,00 | 0,00 | 3,58 | 0,00 | 0,00 | 8,09 | 0,03 | 0,03 |
| 28 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 6,04 | 0,00 | 0,00 | 5,32 | 11,37 | 0,04 | 0,03 |
| 29 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,47 | 12,84 | 0,00 | 0,00 | 0,00 | 0,00 | 13,31 | 0,04 | 0,03 |
| 30 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 8,35 | 0,00 | 0,00 | 0,00 | 0,00 | 8,35 | 0,03 | 0,03 |
| 31 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 10,82 | 10,82 | 0,04 | 0,02 |
| 32 | 0,00 | 0,00 | 11,21 | 6,51 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 17,72 | 0,06 | 0,02 |
| 33 | 0,00 | 0,00 | 7,23 | 1,87 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 9,10 | 0,03 | 0,02 |
| 34 | 0,00 | 0,71 | 0,00 | 0,00 | 0,00 | 7,97 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 8,68 | 0,03 | 0,01 |
| 35 | 0,00 | 0,83 | 0,00 | 0,00 | 20,50 | 0,00 | 4,52 | 0,00 | 0,00 | 0,00 | 0,00 | 25,85 | 0,09 | 0,01 |
| 36 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 12,45 | 0,00 | 0,00 | 0,00 | 0,00 | 12,45 | 0,04 | 0,01 |
| 37 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 14,20 | 0,00 | 0,00 | 0,00 | 14,20 | 0,05 | 0,01 |
| 38 | 1,70 | 0,00 | 0,00 | 9,94 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 11,64 | 0,04 | 0,01 |
| 39 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 40 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 41 | 5,30 | 5,22 | 15,07 | 2,54 | 0,00 | 0,00 | 0,00 | 8,28 | 8,24 | 0,00 | 11,83 | 56,49 | 0,19 | 0,00 |
| 42 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 43 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 44 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 6,18 | 19,57 | 0,00 | 0,77 | 26,52 | 0,09 | 0,00 |
| 45 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 46 | 0,11 | 0,00 | 0,00 | 0,00 | 18,01 | 15,86 | 5,04 | 17,49 | 25,69 | 13,82 | 9,12 | 105,13 | 0,35 | 0,00 |
| 47 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 48 | 1,22 | 0,00 | 0,00 | 0,00 | 0,00 | 1,47 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 2,69 | 0,01 | 0,00 |
| 49 | 6,06 | 2,47 | 0,00 | 0,00 | 0,00 | 0,00 | 5,80 | 0,00 | 0,00 | 0,00 | 0,00 | 14,33 | 0,05 | 0,00 |
| 50 | 0,00 | 2,46 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 1,59 | 0,00 | 0,00 | 0,00 | 4,05 | 0,01 | 0,00 |
| **Mean** | **0,84** | **1,25** | **1,30** | **1,09** | **1,70** | **1,15** | **1,82** | **2,24** | **2,16** | **1,16** | **1,34** | **16,04** | **0,053** | **0,053** |
| **Rate** | **0,060** | **0,089** | **0,053** | **0,045** | **0,061** | **0,041** | **0,065** | **0,064** | **0,062** | **0,033** | **0,038** | **0,053** | | |

**4.2.4.4. Calculating Reinsurance Premiums and Capital Adequacy.** Issuing insurance companies often transfer part of their risk to reinsurance companies because they do not have adequate capital reserves. Issuing companies pay reinsurers a premium to accept this risk. The structure of The transfer can be through a co-pay, a tranche, or a combination of the two.

A co-pay arrangement occurs when liability and indemnity payments are split proportionally. Consider the previous example, where the total liability was 301 million and the required capital was 56.9 million. If the primary insurer has only 10 million of capital to dedicate to crop insurance, then the primary insurer only has 17.6% (10 million/56.9 million) of the required capital. Therefore, the primary insurer should retain(at most) 17.6% of the risk (or 53 million of 301 million) and transfer the remainder (248 million) to another capable risk bearer, usually a reinsurer, who would accept 82.4% of the risk. The premium paid to the reinsurer to absorb this risk would be the pure risk rate of 5.3% plus a load multiplied by the reinsurer liability of 82.4% of 301: 248 million. The pure risk rate is the mean indemnity (16.04 in Table 8) divided by total liability. Indemnities would then be proportional to the risk incurred. In this case, the primary insurer would pay 17.6% of the indemnities and the reinsurer would pay the remaining 82.4%.

In a tranche system, indemnity payment responsibility is hierarchical in that all indemnities are paid by one party up to a specified amount. In many cases, the initial amount is paid by the primary insurer. Once the primary insurer reaches a specific maximum, then a reinsurer is responsible for indemnities beyond that limit. Again, assume that the primary insurer has 10 million in dedicated capital. The first liability tranche is assumed by the primary insurer up to 10 million. The remaining 291 million of liability (or the second tranche) is assumed by the reinsurer. If indemnities total 8 million, then all of these will be paid by the primary insurer. If indemnities are 15 million, then the primary insurer pays the first tranche of 10 million and the reinsurer pays the remaining 5 million. The total capital required is still 56.9 million, but the primary insurer would maintain only 10 million while the reinsurer would be required to maintain 46.9 million.

In terms of capital adequacy, reinsurers are usually evaluated differently than primary insurers, primarily because reinsurers are regulated similarly to publically traded corporations and are paid premiums to absorb this risk. In Table 9, the indemnities for the primary insurer and the reinsurer are calculated for each of the fifty simulated years. The resulting $E[I]$ as a proportion of total liability is 2.1% for the primary insurer (the first tranche) and 3.2% for the reinsurer (the second tranche). Table 9 shows that the reinsurer would make an indemnity payment in twenty-three out of fifty years. As more liability is retained by a primary insurer, the frequency and severity of indemnity payments made by reinsurers declines, which reduces reinsurance costs and loads. However, increasing the primary insurer's liability requires increases in capital.

Table 9. Primary and Reinsurer Indemnities as a Proportion of Total Liability.

| Obs | Indemnities as a Proportion of Total Liability | Indemnities as a Proportion of Total Liability | Indemnities as a Proportion of Total Liability |
|---|---|---|---|
| 1 | 0,0332 | 0,2526 | 0,3160 |
| 2 | 0,0332 | 0,0419 | 0,2526 |
| 3 | 0,0332 | 0,0569 | 0,1823 |
| 4 | 0,0332 | 0,0515 | 0,1727 |
| 5 | 0,0000 | 0,0000 | 0,1544 |
| 6 | 0,0000 | 0,0000 | 0,0797 |
| 7 | 0,0211 | 0,0000 | 0,0569 |
| 8 | 0,0270 | 0,0000 | 0,0549 |
| 9 | 0,0121 | 0,0000 | 0,0527 |
| 10 | 0,0000 | 0,0000 | 0,0515 |
| 11 | 0,0255 | 0,0000 | 0,0419 |
| 12 | 0,0332 | 0,0405 | 0,0405 |
| 13 | 0,0210 | 0,0000 | 0,0346 |
| 14 | 0,0000 | 0,0000 | 0,0256 |
| 15 | 0,0220 | 0,0000 | 0,0144 |
| 16 | 0,0075 | 0,0000 | 0,0139 |
| 17 | 0,0332 | 0,0346 | 0,0110 |
| 18 | 0,0328 | 0,0000 | 0,0081 |
| 19 | 0,0332 | 0,1727 | 0,0078 |
| 20 | 0,0080 | 0,0000 | 0,0055 |
| 21 | 0,0332 | 0,1823 | 0,0045 |
| 22 | 0,0332 | 0,0797 | 0,0027 |
| 23 | 0,0000 | 0,0000 | 0,0027 |
| 24 | 0,0332 | 0,0027 | 0,0000 |
| 25 | 0,0332 | 0,0078 | 0,0000 |
| 26 | 0,0003 | 0,0000 | 0,0000 |
| 27 | 0,0269 | 0,0000 | 0,0000 |
| 28 | 0,0332 | 0,0045 | 0,0000 |
| 29 | 0,0332 | 0,0110 | 0,0000 |
| 30 | 0,0277 | 0,0000 | 0,0000 |
| 31 | 0,0332 | 0,0027 | 0,0000 |
| 32 | 0,0332 | 0,0256 | 0,0000 |
| 33 | 0,0302 | 0,0000 | 0,0000 |
| 34 | 0,0288 | 0,0000 | 0,0000 |
| 35 | 0,0332 | 0,0527 | 0,0000 |
| 36 | 0,0332 | 0,0081 | 0,0000 |
| 37 | 0,0332 | 0,0139 | 0,0000 |
| 38 | 0,0332 | 0,0055 | 0,0000 |
| 39 | 0,0000 | 0,0000 | 0,0000 |
| 40 | 0,0000 | 0,0000 | 0,0000 |
| 41 | 0,0332 | 0,1544 | 0,0000 |
| 42 | 0,0000 | 0,0000 | 0,0000 |
| 43 | 0,0000 | 0,0000 | 0,0000 |
| 44 | 0,0332 | 0,0549 | 0,0000 |
| 45 | 0,0000 | 0,0000 | 0,0000 |
| 46 | 0,0332 | 0,3160 | 0,0000 |
| 47 | 0,0000 | 0,0000 | 0,0000 |
| 48 | 0,0089 | 0,0000 | 0,0000 |
| 49 | 0,0332 | 0,0144 | 0,0000 |
| 50 | 0,0135 | 0,0000 | 0,0000 |
| **Mean** | **0,0215** | **0,0317** | **0,0317** |

## 4.3. Reinsurance Loads

In the example above, the pure risk premium equals 5.3% of the 301 million in total liabilities (Table 9). These values are used with a primary insurer's available equity capital to determine reinsurance costs. For example, assume that three tranches are to be developed and each is loaded differently. The first is the responsibility of a primary insurer with 10 million in equity capital. To calculate loading factors, a premium multiple is first calculated by dividing the available equity by the product of the pure risk rate and total liability (i.e., 10 million/ [0.053 * 301 million]), which results in 0.623. Consequently, the available primary insurer capital is considered adequate to cover indemnities up to 0.623 multiplied by the total premium collections in the first tranche. Table 10 illustrates that the pure risk rate in the first tranche equals 0.023.

Assume that the first tranche of 0.623 premium multiple of liability is not retained by the primary insurer but is ceded to a reinsurer. Reinsurers will usually assign a heavy load of, say, 50% to this first tranche. Multiplying the pure risk rate in the first tranche by (1+0.50) yields a loaded rate of 0.035 (Table 11). The frequency of indemnities in the first tranche is obtained by dividing 39 by 50 (Table 10) to yield 0.78 (Table 11). The severity of indemnities in the first tranche is calculated by dividing the tranche pure risk rate by the frequency which yields 0.0301. This tranche is heavily loaded because the reinsurer will bear high transaction costs if it must accept the risk associated with this frequency of indemnity payments. Only eleven farms are used in this example. If the number of farms were much larger (and more realistic), then frequency would approach 100%.

Assume that two additional tranches are to be established. The second tranche is responsible for indemnities that occur between 62.3% and 400% of risk premiums, while the third tranche pays indemnities that occur in excess of four multiplied by risk premiums. This illustrates the case where one reinsurer may be responsible for the second tranche, and a second reinsurer serves in a stop-loss capacity. Table 11 shows that the pure risk rate for the second tranche is 0.0255. Such tranches often carry a lower load of, say, 20%. In this case, the loaded rate is 0.0307 with a frequency of 0.26 and a severity of 0.098 (Table 11).

Table 10 indicates that the pure risk rate for the third tranche is small (0.0042) primarily because its frequency is only 0.06 (Table 11). However, Table 11 also indicates that the severity in the third tranche is 0.07. Such tranches are often heavily loaded (e.g., 80%) and result in a loaded rate for the third tranche of 0.0076 (Table 11). The reason this last tranche is heavily loaded is illustrated in Figure 7. Distribution of indemnities has a long right-hand tail. Hence, although the probability of indemnities exceeding 4 times the premiums is small, the potential large but infrequent indemnities generate a risk that must be considered by reinsurers.

Table 12 also indicates that the total loaded rate is 0.0735 across the three tranches. The average loaded rate for the three tranches is calculated by dividing the total loaded rate (0.0735) by the pure risk rate (0.053) and subtracting one from the quotient (0.379). The average load for tranches 2 and 3 is calculated by summing the loaded rates for each (0.0307 + 0.0076) and then dividing this by the sum of the pure risk rate for the two tranches (0.0255 + 0.0042). Finally, one is subtracted from the quotient to yield an average load for tranche 2 and 3 of 0.284. The average loaded rate in tranche 2 and 3 is much smaller than the total loaded rate.

Table 11 provides the information needed to calculate the cost to a primary insurer of transferring risk to reinsurers depending upon the form of reinsurance. For example, assume that a Dollar One co-pay is selected. In this case, the primary insurer would retain 53 million in liability and transfer 248 million. The premium for this transfer would be calculated by multiplying the total load rate (0.0735) by the liability being transferred (248 million) which yields 18.235 million. This premium includes a load of 5.018 million, which is calculated by multiplying the pure risk rate (0.053) by the liability being transferred (248 million) before subtracting from the total premium of 18.235 million.

In a tranche situation, the primary insurer would retain 10 million in liability and transfer the remaining 291 million. The premium costs of this transaction are calculated by multiplying the sum of tranche two and three loaded rates (0.0307 + 0.0076) by the total liability (301 million). In this case, the total premium is 11.53 million, of which 2.85 million is the load. The premium for transferring 291 million is much lower for the tranche system than transferring 248 million for a co-pay because the former includes a deductible while the latter does not. In addition, the risk premiums are expected indemnity payouts while the loads are expected net added costs. Therefore, the primary insurer's cost reduction of the tranche system relative to a co-pay arrangement is the difference between 5.01 million and 2.85 million (2.16 million). In effect, the cost reductions to the primary insurer occur because of reductions in the frequency of reinsurance indemnity payments, which lowers reinsurer transaction costs.

Table 10. Sorted Proportions of Premium Risk By Tranche

| Obs | Tranche 1 | Tranche 2 | Tranche 3 |
|---|---|---|---|
| 1 | 0,0332 | 0,1800 | 0,1361 |
| 2 | 0,0332 | 0,1800 | 0,0726 |
| 3 | 0,0332 | 0,1800 | 0,0024 |
| 4 | 0,0332 | 0,1727 | 0,0000 |
| 5 | 0,0332 | 0,1544 | 0,0000 |
| 6 | 0,0332 | 0,0797 | 0,0000 |
| 7 | 0,0332 | 0,0569 | 0,0000 |
| 8 | 0,0332 | 0,0549 | 0,0000 |
| 9 | 0,0332 | 0,0527 | 0,0000 |
| 10 | 0,0332 | 0,0515 | 0,0000 |
| 11 | 0,0332 | 0,0419 | 0,0000 |
| 12 | 0,0332 | 0,0405 | 0,0000 |
| 13 | 0,0332 | 0,0346 | 0,0000 |
| 14 | 0,0589 | 0,0000 | 0,0000 |
| 15 | 0,0476 | 0,0000 | 0,0000 |
| 16 | 0,0472 | 0,0000 | 0,0000 |
| 17 | 0,0442 | 0,0000 | 0,0000 |
| 18 | 0,0413 | 0,0000 | 0,0000 |
| 19 | 0,0411 | 0,0000 | 0,0000 |
| 20 | 0,0387 | 0,0000 | 0,0000 |
| 21 | 0,0378 | 0,0000 | 0,0000 |
| 22 | 0,0360 | 0,0000 | 0,0000 |
| 23 | 0,0360 | 0,0000 | 0,0000 |
| 24 | 0,0328 | 0,0000 | 0,0000 |
| 25 | 0,0302 | 0,0000 | 0,0000 |
| 26 | 0,0288 | 0,0000 | 0,0000 |
| 27 | 0,0277 | 0,0000 | 0,0000 |
| 28 | 0,0270 | 0,0000 | 0,0000 |
| 29 | 0,0269 | 0,0000 | 0,0000 |
| 30 | 0,0255 | 0,0000 | 0,0000 |
| 31 | 0,0220 | 0,0000 | 0,0000 |
| 32 | 0,0211 | 0,0000 | 0,0000 |
| 33 | 0,0210 | 0,0000 | 0,0000 |
| 34 | 0,0135 | 0,0000 | 0,0000 |
| 35 | 0,0121 | 0,0000 | 0,0000 |
| 36 | 0,0089 | 0,0000 | 0,0000 |
| 37 | 0,0080 | 0,0000 | 0,0000 |
| 38 | 0,0075 | 0,0000 | 0,0000 |
| 39 | 0,0003 | 0,0000 | 0,0000 |
| 40 | 0,0000 | 0,0000 | 0,0000 |
| 41 | 0,0000 | 0,0000 | 0,0000 |
| 42 | 0,0000 | 0,0000 | 0,0000 |
| 43 | 0,0000 | 0,0000 | 0,0000 |
| 44 | 0,0000 | 0,0000 | 0,0000 |
| 45 | 0,0000 | 0,0000 | 0,0000 |
| 46 | 0,0000 | 0,0000 | 0,0000 |
| 47 | 0,0000 | 0,0000 | 0,0000 |
| 48 | 0,0000 | 0,0000 | 0,0000 |
| 49 | 0,0000 | 0,0000 | 0,0000 |
| 50 | 0,0000 | 0,0000 | 0,0000 |
| **Mean** | **0,0235** | **0,0256** | **0,0042** |
| **Count** | **39** | **13** | **3** |

**Table 11.** Loads By Tranche

| Item | Tranche 1 | Tranche 2 | Tranche 3 | Total |
|---|---|---|---|---|
| Rate Multiple | 0.0 - 0.6234 | 0.6234 - 4.0 | > 4.0 | |
| Tranche Pure Risk Rate | 0,0235 | 0,0256 | 0,0042 | |
| Load | 0,5 | 0,2 | 0,8 | |
| Loaded Rate | 0,0352 | 0,0307 | 0,0076 | 0,0735 |
| Average Total Load | | | | 0,3797 |
| Average Load Tranche 2 and 3 | | | | 0,2850 |
| Frequency | 0,78 | 0,26 | 0,06 | |
| Severity | 0,0301 | 0,0984 | 0,0704 | |

# 5.0. Insurance Company Pools

It is not uncommon for agricultural insurance companies to develop pools among themselves to gain from scale economies and other benefits of large groups. Smaller insurance companies (those with less than several billion dollars in liability) cannot sufficiently spread the cost of certain activities to be efficient or effectively manage risk. To avoid confusion, these actions are different from the risk rating pools where producers are grouped into homogeneous risk groups for purposes of actuarial accuracy. Insurance pools may provide insurance companies with a variety of efficiencies, including:

1. Data management and accounting services;
2. Development of standardized insurance products;
3. Actuarial and underwriting services;
4. Personnel training, including adjusters and agents;
5. Standardized processes, procedures, and products;
6. Joint risk absorption;
7. Reinsurance;
8. Program education and advertisement;
9. Regulation and interaction with government agencies;
10. Facilitation of government subsidies and related monitoring.

Insurance company pools provide services desired by participating insurance companies or, perhaps, mandated by governments. These pools vary widely in the services provided. The power of insurance company pools also varies widely, ranging from those functioning as quasi-government agencies to those having little coercive power so that they merely provide services to each company based on fee-for-service business actions. Ultimately, insurance company pools can provide benefits through scale economies, standardization, reductions in reinsurance transaction costs, and lower reinsurance loads.

## 5.1. Pooling Arrangements

Insurance company pooling arrangements reduce average risk exposures by maximizing the benefits of diversification and standardization. In addition to gains from scale economies, improvements to self-regulation interactions with regulatory agencies may be captured.

**5.1.1. Benefits and Costs of Insurance Company Pools.** Insurance pools have several advantages, including:

1. Gains from scale economies through reductions in development and administrative costs and increased standardization of insurance programs and products;
2. Increased safety and soundness as a result of expansions in equity capital reserves;
3. Improved customer trust and loyalty because of standardized loss adjustment procedures and product servicing;
4. Lower insurance costs through reductions in reinsurance premiums resulting from standardization, scale, and diversification;
5. Better arrangements with governments that provide stop-loss and other risk-sharing activities.

In addition, insurance companies that participate in pooling activities often create value in a variety of ways. For example, such companies often have vested interests in providing functions related to:

1. Conducting risk analyses based on historical yield data and historical loss data;
2. Developing insurance programs;
3. Increasing product standardization;
4. Improving underwriting activities, including the defining insured risks, terms of insurance contracts, loss adjustment procedures, rating, reporting procedures, data security, and documentation;
5. Managing networks of loss adjusters;
6. Improving claims procedures and providing indemnity payments schedules;
7. Coordinating internal reinsurance among participating companies;
8. Arranging government reinsurance activities and other forms of risk sharing;
9. Coordinating interactions with government regulations;
10. Providing centralized data-backup procedures;
11. Organizing insurance program advocacy and producer education;
12. Developing uniform procedures relative to policy sale closing dates, required inspections, timeliness of indemnity payments, and communications.

While insurance company pools can create value, they also increase the complexity of insurance programs and generate additional costs. A variety of legal arrangements must be developed, including clear delineation of risks being transferred, responsibility for indemnity payments, premium sharing, reinsurance costs, program monitoring, and other operational costs.

**5.1.2. Binding Agreements and Enforcement.** Insurance pools require various agreements to be developed and enforced. For example, one insurance company's book-of-business could result in large indemnity payments in a given year, while other pool companies' book-of-businesses result in only minor indemnity obligations. Pooling arrangements and agreements may be established so that those in the pool assist with providing indemnity payments for pool members who have large indemnity obligations in any given year. One positive aspect of these arrangements, however, is that capital adequacy for any single insurance company is reduced.

## 5.2. Cash-Flow Model

Cash-flow models are used to predict future cash-flow needs and evaluate alternatives. As an example, a cash-flow model is developed to illustrate issues related to insurance company pools and their effect on capital adequacy. The cash-flow model highlights gains that can be realized from insurance company pools.

**5.2.1. Model Development.** Assume that two insurance companies (A and B) provide yield insurance in a region. The average yield for producers in the region is 2 metric tons/hectare, and each produer selects 65% coverage. The two insurers cover a total of 100 hectares. Primary Insurer A has 60% of the market (60 hectares) and Primary Insurer B has 40% (40 hectares). Table 12 presents per ton prices, total liability, total premiums, and total indemnities for the years 1992–2012 for the combined insurance activity of Primary Insurers A and B. Total premiums are calculated from the data by summing the pure risk rate of 0.092 (average annual total indemnities divided by average annual liability) and a load of 30%.

Table 13 presents the cash-flow situation for Primary Insurer A, which has 60% of the market. Cash inflows are represented by premiums and any reinsurance payments that may be forthcoming. The retained risk of Primary Insurer is assumed to be one multiplied by the pure risk premium. Hence, reinsurance payments are received when indemnities exceed retained risk.

Cash outflows are represented by indemnity payments and normal business operating costs. Operating costs are assumed to be 15% of the pure risk rate multiplied by liability. The costs of belonging to insurance company pools are also a component of cash outflow. Pool costs are assumed to be 1.5% of the pure risk rate multiplied by liability. Reinsurance premiums are a cash outflow and are calculated by multiplying Primary Insurer A's reinsurance rate by its liability. The reinsurance rate is the quotient of Primary Insurer A's pure risk rate of 0.042 after loading it by 18% and the combined average crop liability ($10,294) presented in Table 12. The pure risk rate

Table 12. Combined Crop Insurance Example

| Year | Price Per Metric Ton | Total Liability | Total Premiums | Total Indemnities |
|---|---|---|---|---|
| 1992 | 5,40 | 10 530 | 1 262 | 420 |
| 1993 | 2,91 | 5 672 | 679 | 1 661 |
| 1994 | 5,11 | 9 968 | 1 194 | 65 |
| 1995 | 4,19 | 8 165 | 978 | 50 |
| 1996 | 5,63 | 10 970 | 1 314 | 120 |
| 1997 | 4,68 | 9 118 | 1 092 | 20 |
| 1998 | 7,89 | 15 387 | 1 843 | 71 |
| 1999 | 4,80 | 9 360 | 1 121 | 5 050 |
| 2000 | 7,57 | 14 756 | 1 768 | 1 137 |
| 2001 | 3,23 | 6 303 | 755 | 289 |
| 2002 | 4,27 | 8 320 | 997 | 901 |
| 2003 | 4,51 | 8 796 | 1 054 | 92 |
| 2004 | 5,51 | 10 740 | 1 287 | 2 275 |
| 2005 | 4,66 | 9 092 | 1 089 | 85 |
| 2006 | 4,52 | 8 815 | 1 056 | 1 255 |
| 2007 | 5,78 | 11 271 | 1 350 | 1 586 |
| 2008 | 4,47 | 8 717 | 1 044 | 318 |
| 2009 | 6,87 | 13 392 | 1 604 | 1 126 |
| 2010 | 6,33 | 12 337 | 1 478 | 2 041 |
| 2011 | 6,38 | 12 447 | 1 491 | 30 |
| 2012 | 6,16 | 12 017 | 1 440 | 1 329 |
| **Average** | **5,28** | **10 294** | **1 233** | **949** |

for Primary Insurer A is calculated based on the sum of average annual reinsurance payments made to this primary insurer and Primary Insurer B (see below), assuming that the two companies do not pool their liabilities prior to purchasing reinsurance. Net cashflow represents the difference between cash inflows and cash outflows in any given year. Average annual net cash inflow for Primary Insurer A is $45.

Table 13. Cash Flow of Primary Insurer A

| Year | Liability | Cash Inflow | | Cash Outflow | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Premiums | Reinsurance Payment | Indemnity | Operational Cost | Pool Cost | Reinsurance Premium | Net Cashflow |
| 1992 | 6 318 | 757 | - | 300 | 87 | 9 | 316 | 45 |
| 1993 | 3 403 | 408 | 574 | 982 | 47 | 5 | 170 | (222) |
| 1994 | 5 981 | 716 | - | 11 | 83 | 8 | 299 | 316 |
| 1995 | 4 899 | 587 | - | 34 | 68 | 7 | 245 | 234 |
| 1996 | 6 582 | 789 | - | 99 | 91 | 9 | 329 | 261 |
| 1997 | 5 471 | 655 | - | 8 | 76 | 8 | 273 | 291 |
| 1998 | 9 232 | 1 106 | - | 41 | 128 | 13 | 461 | 463 |
| 1999 | 5 616 | 673 | 3 826 | 4 499 | 78 | 8 | 281 | (366) |
| 2000 | 8 854 | 1 061 | - | 561 | 122 | 12 | 442 | (78) |
| 2001 | 3 782 | 453 | - | 114 | 52 | 5 | 189 | 93 |
| 2002 | 4 992 | 598 | 252 | 850 | 69 | 7 | 249 | (325) |
| 2003 | 5 277 | 632 | - | 57 | 73 | 7 | 264 | 231 |
| 2004 | 6 444 | 772 | - | 154 | 89 | 9 | 322 | 198 |
| 2005 | 5 455 | 654 | - | 33 | 75 | 8 | 273 | 265 |
| 2006 | 5 289 | 634 | 428 | 1 062 | 73 | 7 | 264 | (345) |
| 2007 | 6 763 | 810 | - | 522 | 93 | 9 | 338 | (153) |
| 2008 | 5 230 | 627 | - | 226 | 72 | 7 | 261 | 60 |
| 2009 | 8 035 | 963 | - | 681 | 111 | 11 | 401 | (242) |
| 2010 | 7 402 | 887 | 769 | 1 656 | 102 | 10 | 370 | (482) |
| 2011 | 7 468 | 895 | - | 11 | 103 | 10 | 373 | 397 |
| 2012 | 7 210 | 864 | - | 90 | 100 | 10 | 360 | 304 |
| **Average** | **6 176** | **740** | **279** | **571** | **85** | **9** | **309** | **45** |

Table 14 shows similar information for Primary Insurer B, which has 40% of the market. Again, reinsurance premiums represent a cash outflow and are calculated assuming a pure risk rate of 0.042 with an 18% load. The pure risk rate is the same as that for Primary Insurer A, assuming that the two companies do not pool their liabilities prior to purchasing reinsurance. Average annual net cash inflows total $5.

**Table 14.** Cash Flow of Primary Insurer B

| Year | Liability | Cash Inflow | | Cash Outflow | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Premiums | Reinsurance Payment | Indemnity | Operational Cost | Pool Cost | Reinsurance Premium | Net Cashflow |
| 1992 | 4 212 | 505 | - | 120 | 58 | 6 | 210 | 110 |
| 1993 | 2 269 | 272 | 407 | 679 | 31 | 3 | 113 | (148) |
| 1994 | 3 987 | 478 | - | 54 | 55 | 6 | 199 | 164 |
| 1995 | 3 266 | 391 | - | 16 | 45 | 5 | 163 | 162 |
| 1996 | 4 388 | 526 | - | 21 | 61 | 6 | 219 | 219 |
| 1997 | 3 647 | 437 | - | 12 | 50 | 5 | 182 | 187 |
| 1998 | 6 155 | 737 | - | 30 | 85 | 9 | 307 | 306 |
| 1999 | 3 744 | 449 | 103 | 551 | 52 | 5 | 187 | (244) |
| 2000 | 5 902 | 707 | - | 576 | 82 | 8 | 295 | (253) |
| 2001 | 2 521 | 302 | - | 175 | 35 | 3 | 126 | (37) |
| 2002 | 3 328 | 399 | - | 51 | 46 | 5 | 166 | 131 |
| 2003 | 3 518 | 421 | - | 35 | 49 | 5 | 176 | 157 |
| 2004 | 4 296 | 515 | 1 606 | 2 121 | 59 | 6 | 215 | (280) |
| 2005 | 3 637 | 436 | - | 52 | 50 | 5 | 182 | 147 |
| 2006 | 3 526 | 422 | - | 193 | 49 | 5 | 176 | (1) |
| 2007 | 4 508 | 540 | 524 | 1 064 | 62 | 6 | 225 | (294) |
| 2008 | 3 487 | 418 | - | 92 | 48 | 5 | 174 | 99 |
| 2009 | 5 357 | 642 | - | 445 | 74 | 7 | 268 | (152) |
| 2010 | 4 935 | 591 | - | 385 | 68 | 7 | 247 | (115) |
| 2011 | 4 979 | 596 | - | 19 | 69 | 7 | 249 | 253 |
| 2012 | 4 807 | 576 | 664 | 1 239 | 66 | 7 | 240 | (313) |
| **Average** | **4 118** | **493** | **157** | **378** | **57** | **6** | **206** | **5** |

Table 15 presents the net cashflow for a primary insurance pool consisting of Primary Insurers A and B. Cash inflows are simply the sum of the columns presented in Tables 13 and 14. Likewise, indemnity, operational cost, and pool costs are also the sum of their respective columns in Tables 13 and 14. However, the pool's reinsurance premium is smaller than the sum of the premiums paid individually by Primary Insurers A and B because the pool's pure risk rate of 0.032 is lower than the individual pure risk reinsurance rate of 0.042. The pool rate is lower because of the diversification across Primary Insurers A and B book-of-business. The pool pure risk rate is calculated by dividing the average annual pool reinsurance payment ($328) by the average annual combined liability ($10,294). This results in an average annual net cash inflow of $69 for the pool, which is larger than the sum of the average annual net cash inflow of the two insurers ($45 + $5 = $50). However, reinsurance at the pool level requires pool participants to agree that they will each help those pool members who incur unusually large indemnity obligations. Alternatively, each pool member must retain higher capital requirements.

In addition, these savings may be even larger, as a reinsurer may load the pure risk rate at a lower level in pooled arrangements because of reductions in transaction costs and improved standardization.

Table 15. Cash Flow of Both Primary Insurers

| Year | Liability | Cash Inflow | | Cash Outflow | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Premiums | Reinsurance Payment | Indemnity | Operational Cost | Pool Cost | Reinsurance Premium | Net Cashflow |
| 1992 | 10 530 | 1 262 | - | 420 | 146 | 15 | 396 | 285 |
| 1993 | 5 672 | 679 | 981 | 1 661 | 78 | 8 | 213 | (300) |
| 1994 | 9 968 | 1 194 | - | 65 | 138 | 14 | 375 | 602 |
| 1995 | 8 165 | 978 | - | 50 | 113 | 11 | 307 | 497 |
| 1996 | 10 970 | 1 314 | - | 120 | 152 | 15 | 413 | 615 |
| 1997 | 9 118 | 1 092 | - | 20 | 126 | 13 | 343 | 591 |
| 1998 | 15 387 | 1 843 | - | 71 | 213 | 21 | 579 | 959 |
| 1999 | 9 360 | 1 121 | 3 929 | 5 050 | 129 | 13 | 352 | (495) |
| 2000 | 14 756 | 1 768 | - | 1 137 | 204 | 20 | 555 | (149) |
| 2001 | 6 303 | 755 | - | 289 | 87 | 9 | 237 | 133 |
| 2002 | 8 320 | 997 | - | 901 | 115 | 12 | 313 | (344) |
| 2003 | 8 796 | 1 054 | - | 92 | 122 | 12 | 331 | 497 |
| 2004 | 10 740 | 1 287 | 988 | 2 275 | 148 | 15 | 404 | (568) |
| 2005 | 9 092 | 1 089 | - | 85 | 126 | 13 | 342 | 524 |
| 2006 | 8 815 | 1 056 | 199 | 1 255 | 122 | 12 | 332 | (466) |
| 2007 | 11 271 | 1 350 | 236 | 1 586 | 156 | 16 | 424 | (596) |
| 2008 | 8 717 | 1 044 | - | 318 | 120 | 12 | 328 | 266 |
| 2009 | 13 392 | 1 604 | - | 1 126 | 185 | 19 | 504 | (229) |
| 2010 | 12 337 | 1 478 | 563 | 2 041 | 171 | 17 | 464 | (652) |
| 2011 | 12 447 | 1 491 | - | 30 | 172 | 17 | 469 | 803 |
| 2012 | 12 017 | 1 440 | - | 1 329 | 166 | 17 | 452 | (525) |
| **Average** | **10 294** | **1 233** | **328** | **949** | **142** | **14** | **387** | **69** |

### 5.2.2. Reinsurance Designs

Reinsurance design issues for insurance pools are similar to those faced by individual insurance companies. The central issue is how much risk, if any, an insurance pool desires to cede to a reinsurer versus the amount it is willing to retain. If a pool assumes some indemnity risk, then arrangements must be made for the provision of capital to support that risk. However, if the pool cedes all of the risk to a reinsurer, it can still perform important services by facilitating reinsurance outcomes. Rating procedures for reinsurance with pool involvement follow the same procedures discussed in the Actuarial Manual. The following examples illustrate these issues.

Consider a case in which a pool arranges and negotiates reinsurance with a reinsurance company. The arrangement includes provisions for each primary insurance company to retain a part of the indemnity risk, either as a co-pay or as a tranche. Each company's reinsurance could be developed as a separate book-of-business with reinsurance indemnities paid based on an individual company's indemnity obligation. In this case, the pool acts as a facilitator but bears none of the indemnity risk. However, the combined book-of-business for the pool may be sufficiently large to be considered more valuable to reinsurance companies and could result in lower reinsurance premiums relative to the book-of-business for smaller individual insurance companies.

A second scenario occurs if a pool is established so that the pool's book-of-business is the combination of the amount of risk each individual insurance company cedes to the pool. Each individual company could retain some risk (e.g., one times premiums multiplied by the pure risk rate) with the remainder ceded to the pool. The pool could then obtain reinsurance for aggregate indemnities that exceed one times premiums multiplied by the pure risk premium for the pool's share of the overall risk. In this situation, the pool faces some risk because a single company's indemnities could exceed one timies premiums multiplied by the pure risk premium, while aggregate indemnities across all companies do not exceed the aggregate of one times premiums multiplied by the pure risk premium. In this case, the indemnities would not trigger reinsurance payments. Therefore, the pool must have sufficient capital available capital to make indemnity payments.

**5.2.3. Capital Adequacy.** The calculation of capital adequacy for pools that retain some indemnity risk follows the procedures discussed earlier for primary insurance companies. These calculations are often more complicated and depend on the specifics of reinsurance and risk bearing design of the pool. Parametric or Monte Carlo approaches can be used to determine capital adequacy.

## 5.3. Ukraine Example

A cash-flow simulation model is developed for Ukraine to illustrate the information that insurance pool managers and reinsurers need. The model is used to estimate net cash flows. The model is flexible, allowing a variety of scenarios to be simulated. In this section, the model is used to illustrate net cash flows for two insurance pool scenarios. In the first, the pool is assumed to retain 100% of the pure risk premium. In the second, the pool retains 200% of the pure risk premium.

**5.3.1. Model Assumptions.** The simulation model is based on actual historical winter wheat data for Ukraine from 1992–2011 obtained from its multiple peril crop insurance (MPCI) program. The model is used to forecast cash-flow requirements for the eighteen years between 2012 and 2029.

Historical winter wheat production data include planted area (thousands of hectares) and average yield (tons/hectare). The average annual price is assumed to be $100 UAH/ton.

The pure premium rate is also calculated from the historical data. The model assumes:

- A pure risk premium load of 30%
- Producer participation rates of 50%
- Coverage level of 65%;
- Primary insurer's operational costs as 15% of premiums;
- Insurance pool's operational costs as 1.5% of premiums.

The model simulates the cash-flow effects of risk distribution among four levels of participants: (1) a primary insurance company, (2) an insurance company pool, (3) a private reinsurer, and (4) a government stop-loss program. Indemnities are assumed to be paid in hierarchical tranches.

Annual insurance premium rates, premium collections, and liability are estimated based on the available data and using the above assumptions. These values are used to obtain monthly premium cash inflows and cash outflows (i.e., reinsurance premiums, operational costs and indemnities). The cash flows are simulated on a monthly basis and then aggregated to annual cash-flow requirements.

Reinsurance premiums are calculated dynamically using simulations based on actual historical data. The total reinsurance premium is a function of the magnitude of the liability transferred to the reinsurer. The amount of the liability transferred is a function of the pool's reserve-fund balance and can range from 50% to 90% of total liability.

Operational costs represent the costs incurred by primary insurance companies and the reinsurance pool. These costs are calculated using the assumptions noted above.

Indemnities may be provided by up to four sources in any given year: the primary insurance company, reinsurance pool, private reinsurer, and government stop-loss program. Indemnity payments as a percent of the total liability are calculated from the model for the period of 1992–2011.

Insurance companies must maintain sufficient capital reserves to cover potential indemnities. Indemnity payments in any given year could exceed one or more years of premium cash inflow. Consequently, a substantial portion of this liability is often transferred to a reinsurer. As larger amounts of liability are transferred to a reinsurer, smaller reserves are needed by a primary insurer. However, reinsurance costs also increase with increased levels of risk transfer.

**5.3.2. Model Results.** Figure 9 presents the pool fund's net cash flow using two scenarios: (1) the pool retains 100% of the pure risk premium and (2) the pool retains 200% of the pure risk premium. In both cases, the remaining risk is ceded to a reinsurer. Annual net cash flow is more variable when the pool retains a higher level of the pure risk premium.

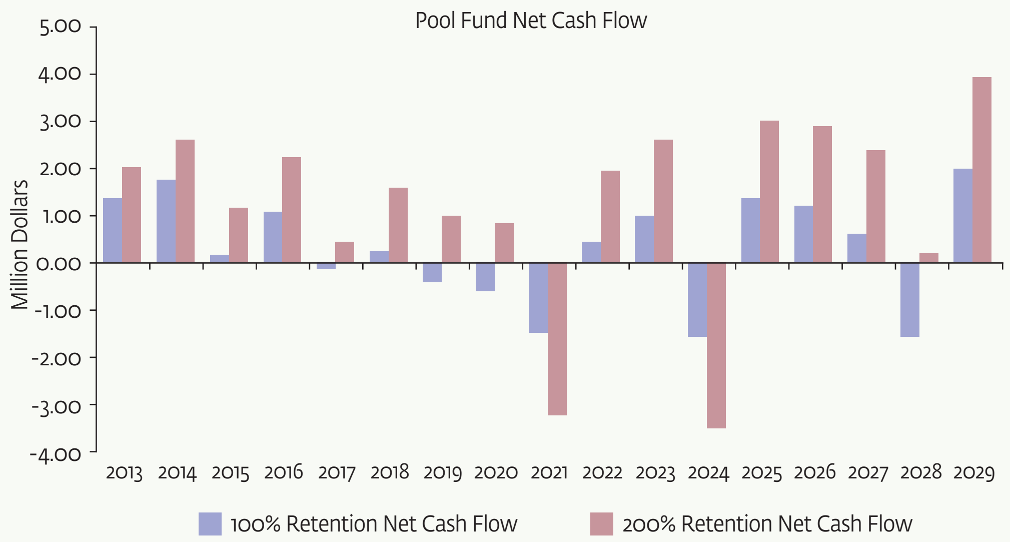


**Figure 9.** Pool Fund Annual Net Cash Flow for 100% and 200% Pure Risk Premium Retention

Figure 10 presents the pool fund's accumulated cash flow under the same scenarios. Accumulated net cash flow is larger when the pool retains 200% of the pure risk premium. However, as illustrated in figure 9, the pool is accepting more risk.

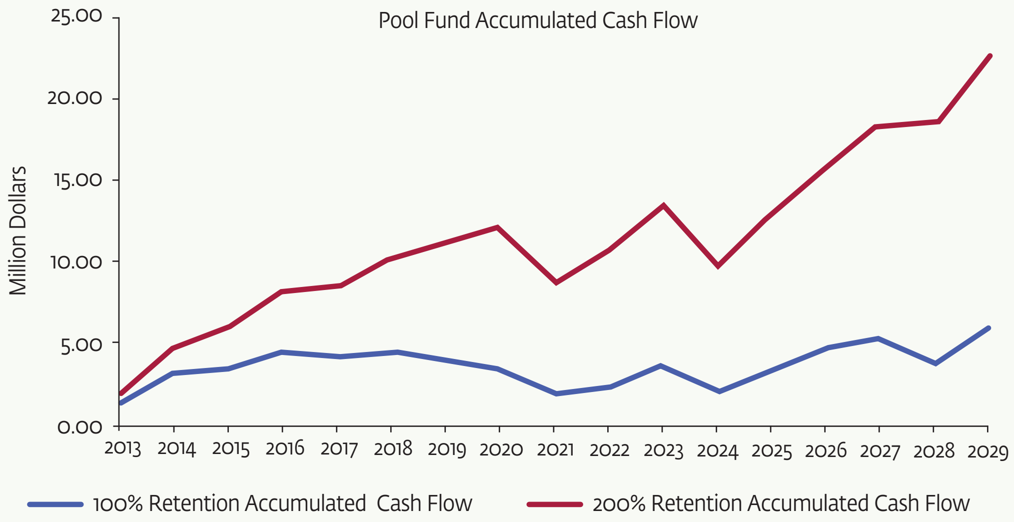


**Figure 10.** Pool Fund Accumulated Net Cash Flow for 100% and 200% Pure Risk Premium Retention

## 5.4. Summary of Insurance Pools

Insurance company pooling arrangements reduce average risk exposure by maximizing diversification and standardization benefits. In addition to gains from scale economies, self-regulatory improvements can create better interactions with regulatory agencies. Pooling can also lower reinsurance costs and reduces capital requirements.

Substantial cooperation among insurance companies within pools is required. While insurance company pools can create value, they also increase the complexity of insurance programs and generate additional costs. A variety of legal arrangements must be developed, including clear delineation of risks being transferred, responsibility for indemnity payments, premium sharing, reinsurance costs, program monitoring, and other operational costs.

# 6.0. Summary

Agricultural production is inherently subject to a variety of risks because management decisions or states-of-nature often generate future outcomes (either favorable or unfavorable) that cannot be predicted with certainty. Some risks are managed through production and financial decision-making while others are simply accepted as business expenses. In addition, some risks can be managed through a variety of contractual and insurance-related products.

On average, financial activities with low levels of risk are associated with lower potential returns. Conversely, high levels of financial risk are generally associated with high expected returns. However, the risk/return trade-off does not mean that accepting high levels of risk *guarantees* higher returns. Rather, high levels of risk provide the *possibility* of high returns and vice versa. Individuals and firms must be compensated for accepting higher levels of risk with at least the potential to receive higher returns.

Risks associated with agricultural production ultimately impact the financial viability and sustainability of farms and ranches. Agricultural production is often coincident with high short-term credit risk because of a combination of high fixed costs, weather variability, disease, and variations in cash receipts. Whether an agricultural producer self-insures or uses formal mechanisms for transferring risk to others, risk is a cost that must be managed effectively.

Agricultural production risks also impact the viability of businesses that supply agricultural credit and insurance services to agricultural producers. Agricultural finance companies must account for potential reductions in debt repayment as a result of agricultural production risks. Hence, they must either maintain adequate capital reserves or pay fees to transfer this risk to other entities.

Although a variety of approaches exist to manage risk, each involves transaction costs and risk premiums paid by those seeking to mitigate risk to those willing to accept additional risk. Transaction costs and risk premiums can be incorporated into: (1) interest rates, (2) insurance, and (3) other instruments.

Loan default risk can be incorporated into operating, intermediate, and real estate loan interest rates. The advantage of incorporating risk premiums into interest rates (rather than other instruments) is that transaction costs are reduced because only two entities (a borrower and a lender) are involved. That is, the costs of risk transfer increase as additional entities are included. However, incorporating risk premiums into interest rates is also problematic. For example, higher interest rates increase the probability of loan default, reduce farm profitability and repayment capacity, and hamper investment in production-expanding technologies.

The availability and use of agricultural insurance reduces credit risk, lowers interest rates, improves repayment capacities, increases credit availability, and reduces financial and business risk. Crop insurance costs, however, can also be substantial.

The pure risk premium component of interest rates in the absence of insurance is exactly equal to an insurance pure risk premium if the policies perfectly insure against loan default perils. As a result, insurance increases business costs only in the sense that it increases transaction costs.

Agricultural credit and insurance firms acquire risk through lending and insurance business practices. However, they must also balance the risk of loan defaults and insurance indemnities while maintaining adequate capital reserves. That is, above-average loan defaults or unexpectedly large indemnity payments require sufficient equity capital to maintain business integrity. As with any business firm, credit and insurance companies must have sufficient capital to manage unexpected cash outflows. Consequently, credit and insurance firms must decide how to manage such risk. Credit and primary insurers often transfer risk to other companies in exchange for a fee. This process is generally termed "reinsurance."

Agricultural production and revenue risks are only partially diversifiable across production sectors and regions. Hence, primary agricultural insurers often cede risk to reinsurers who compile diversified risk portfolios. Reinsurers charge fees to primary insurers in exchange for this risk transfer. Reinsurers are usually large, international companies that are well diversified across regions, countries, and economic sectors. Many governments also provide reinsurance opportunities. In many cases, governments provide stop-loss reinsurance services that support both primary insurers and reinsurers.

Multiple forms of reinsurance exist, and each is defined by the manner in which risks are distributed between a primary insurer and a reinsurer (or reinsurers). In general, these approaches are classified as proportional, nonproportional, or combination agreements. The costs of obtaining reinsurance depend upon pure risk premium rates as well many other factors. In addition, loads charged by reinsurers account for other elements such as servicing costs, program design, program integrity, underwriting issues, political, judicial and legal risks, personnel competency, reputation of involved parties, and the costs of capital. High loads are applied to countries without stable, equitable, and well-developed judicial systems. In addition, many of these situations are also coincident with poorly constructed contractual law and property rights.

Primary insurers develop reinsurance submissions as business proposals for consideration by reinsurers. Reinsurers use these submissions to evaluate their willingness to offer reinsurance and to determine appropriate risk transfers, costs, and loads. Reinsurance companies are also attracted to insurance products that are standardized across areas and countries. These products allow for common program administration, loss adjustment, and accounting.

The safety and soundness of financial institutions has become a leading issue because of the recent global financial crisis. The trade-off between risks and rewards generates difficult decisions regarding conservative versus aggressive financial management strategies. Firms must decide between sacrificing long-term economic growth versus financial risks. Many governments regulate financial institutions, including banks, insurance, securities, thrift/credit unions, and futures markets. Some countries also regulate secondary financial markets and associated service industries such as accounting and auditing firms. Although most financial regulation has historically focused on banking institutions, it has recently spread to other financial entities. The modern era of bank regulation was initiated with the Basel Agreements, which measure credit, market, and operational risk and capital adequacy. Parametric and Monte Carlo simulations are used to evaluate Value-at-Risk.

Insurance company pooling arrangements reduce average risk exposures by maximizing the benefits of diversification and standardization. In addition to gains from scale economies, self-regulation improvements can be developed as well as improved interactions with regulatory agencies. Such pooling can provide benefits through scale economies, standardization, reductions in reinsurance transaction costs, and lower reinsurance loads.

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